1. What complexity class that we have seen previously is equal to $\mathcal{IP}[0]$?

2. Consider the following definition of the class $\mathcal{IP}^*$: a language $L$ is in $\mathcal{IP}^*$ if there exist $(P,V)$ with $V$ running in probabilistic polynomial time such that:
   - If $x \in L$ then $\Pr[(P,V)(x) = 1] \geq 3/4$.
   - If $x \notin L$ then $\Pr[(P,V)(x) = 1] = 0$.

   Prove that $\mathcal{IP}^* = \mathcal{NP}$.

3. Show an AM protocol for the following promise problem:

   $\Pi_Y = \{(\phi, k) \mid \#\text{SAT}(\phi) > 8k\}$
   $\Pi_N = \{(\phi, k) \mid \#\text{SAT}(\phi) < k/8\}$.

   I.e., when $x \in \Pi_Y$ then the prover should be able to convince the verifier with probability 1 while if $x \in \Pi_N$ then the prover should be unable to convince the verifier with probability better than 1/2. (Hint: it will be easier to construct an AM protocol with non-zero completeness error first...). Why doesn’t this show that $\#\mathcal{P} \subseteq \text{AM}$?

4. Sketch an interactive proof system for $\Sigma_2 = \mathcal{NP}^{\mathcal{NP}}$ without relying on the $\mathcal{PSPACE} \subseteq \mathcal{IP}$ result. Hint: use the proof system for co$\mathcal{NP}$ that we showed in class as a black box...

5. Work out in full an execution of the interactive proof system for $\mathcal{PSPACE}$ that we showed in class (assuming an honest prover), for the true statement:

   $\phi = \forall x_1 \exists x_2 : (x_1 \lor \bar{x}_2) \land (\bar{x}_1 \lor x_2)$.

   What is the smallest prime $q$ that the parties can use for this $\phi$ to guarantee soundness error at most 1/2 (recall that the verifier can compute the arithmetization of $\phi$, too...)?