University of Maryland
CMSC652 — Complexity Theory
Professor Jonathan Katz

Homework 2

Due at the *beginning* of class on Sept. 28

I suggest to use LaTeX when typing up your solutions.

1. An undirected graph G can be *k-colored* if each of its vertices can be assigned a "color" in $\{1, \ldots, k\}$ such that no vertices that share an edge have the same color. Let

$$\mathsf{3COL} = \{G : G \text{ can be 3-colored}\}.$$

   Prove that $\mathsf{3COL}$ is $\mathcal{NP}$-complete.

2. Let L be an $\mathcal{NP}$-complete language. Prove that if $L \in \mathsf{co}\mathcal{NP}$ then $\mathcal{NP} = \mathsf{co}\mathcal{NP}$.

3. Prove that Definition 4.19 (the certificate-based definition) yields the same class NL as the definition of NL based on non-deterministic Turing machines.

4. A non-deterministic machine M computes a function $f : \{0,1\}^* \to \{0,1\}^*$ if the following holds:

   - For every x, there exists a computation path such that $M(x)$ accepts.
   - In any computation path on which $M(x)$ accepts, the correct result $f(x)$ is written on the output tape when M halts. (On computation paths where $M(x)$ does not accept, anything may be written on the output tape.)

   Answer the following questions:

   (a) Let $f(G, s, t)$ be the function that outputs a path from s to t in directed graph G (or $\perp$ if there is no path). Show that f can be computed by a non-deterministic log-space machine.

   (b) Show that f can be computed by a deterministic machine in space $O(\log^2 n)$.

   (c) Show that any function that can be computed by a non-deterministic machine in space $s(n)$ can be computed by a deterministic machine in space $O(s(n)^2)$.

5. Barak-Arora, Exercise 4.5. (*Hint*: reduce 2SAT to $\overline{\text{CONN}}$.)