1. This problem tests your understanding of the formal language operations defined in lecture, which are necessary for understanding regular expressions.

Consider the languages or sets of strings $A = \{a, aa, aaa\}$ and $B = \{bb\}$. Show the languages denoted by each of the following:

a. $A^1$

b. $A^2$

c. $A \cup A^2$

d. $A^*$

e. $B^1$

f. $B^2$

g. $B^3$

h. $B \cup B^2 \cup B^3$

i. $B^*$

j. $(AB)^2$

k. $(A \cup B)^2$

2. Write a regular expression which describes or recognizes each of the following languages. Your regular expressions should use only the three regular expression operations concatenation, alternation, and Kleene closure, as defined in lecture. Use $\epsilon$ to denote the empty string. The underlying alphabet for each part is $\Sigma = \{a, b\}$.

The notation $\#a(w)$ is used below to refer to the number of $a$’s occurring in the string $w$. For example, $\#a(bbaba) = 2$.

a. $\{w \mid w \text{ begins with } abab \}$

b. $\{w \mid w \text{ ends with } abab \}$

c. $\{w \mid w \text{ begins with } ab \text{ and ends with } ba \}$

Note: The string $aba$ is in this language.

d. $\{w \mid \#a(w) \mod 5 = 2 \}$

Recall that $i \mod k = j$ if and only if $i - j$ is divisible by $k$.

e. $\{w \mid \#a(w) \text{ is even or } |w| \text{ is even} \}$

f. $\{w \mid aaa \text{ is a substring of } w \}$

g. $\{w \mid aaa \text{ is not a substring of } w \}$
3. Consider the following language:
\[ \{ w \mid w \in \{ 0, 1 \}^* \text{ and } w \text{ contains an even number of 0s, and } w \text{ does not contain three consecutive 1s } \} \]
Determine whether each of the following regular expressions correctly describes or recognizes this language or not. Identify why each incorrect regular expression is wrong—give a string which the regular expression doesn’t give the right results for, and identify what result the regular expression should give for that string, and what result it actually gives.

a. \( (0 (\epsilon | 11) 0)^* \)

b. \( ((0 (\epsilon | 11) 0)^* \mid 1 \mid 11)^* \)

c. \( ((0 (\epsilon | 11) 0)^* \mid 1 \mid 11)^* \)

d. \( ((\epsilon | 11) 0 (\epsilon | 11) 0 (\epsilon | 11) 0)^* \mid 1 \mid 11)^* \)

e. \( ((\epsilon | 11) 0 (\epsilon | 11) 0)^* \mid 1 \mid 11)^* \)

f. \( (\epsilon | 11) (0 (\epsilon | 11) 0)^* (\epsilon | 11) \)

g. \( (\epsilon | 11) 0 (\epsilon | 11) 0)^* (\epsilon | 11) \)

h. \( ((0 \mid 01 \mid 011 \mid 10 \mid 101 \mid 1011 \mid 110 \mid 1101 \mid 11011) 0)^* (\epsilon | 11) \)