1. Give regular expressions for the following languages. You should only use formal regular expressions as defined on the slides.
   (a) Binary strings ending in 01
   (b) Decimal integers divisible by 5. Initial 0’s are not allowed (e.g., 015 is invalid, but 15 is a valid string; 0 is also a valid string).
   (c) Ruby identifiers. (You’ll need to find out what valid Ruby identifiers are on your own.)
   (d) Binary strings consisting of either an odd number of 0s or an odd number of 1s.
   (e) Binary strings containing the sequence 101 embedded within it.
   (f) Binary strings that do not contain the sequence 101 embedded within it.

2. Give DFA for:
   (a)-(f) in (2) above
   (g) Decimal integers divisible by 9. (Hint: Look up casting out 9s) The DFA for this problem might be easier written down in formal notation (with a table for the transition function) rather than drawing a picture.

3. Give the regular expression that accepts the same set as the following NFA:

4. Given the set \( L = \{ a^{3m}b^{2n} \mid m \geq 0; n \geq 0 \} \)
   (a) Describe \( L \) as a regular expression.
   (b) Give a DFA that recognizes this set.
   (c) Does the regular expression \( a^*b^* \) generate \( L \) since every string in \( L \) can also be generated by \( a^*b^* \)? Explain.

5. Let \( S \) be a set recognized by a DFA. Prove that \( S^R \) (S-reversed, i.e., every string in \( S \) written in reverse order) is recognized by a DFA.

6. Let \( R \) be a set of even length strings generated by a DFA. Let \( R^{1/2} \) be the “first halves” of all strings in \( R \). That is, if \( w \) is in \( R \) and \( w \) is of length \( 2n \), then the first \( n \) symbols of \( w \) is recognized by \( R^{1/2} \). Show that \( R^{1/2} \) is recognized by a DFA. (Hint 1: Solve problem 5 first. Hint 2: Run a DFA backward and forward at the same time.)

Note: This is a hard problem. Few will probably get this one correct, but everyone should attempt it. Hint 2 should make the problem a bit easier to solve.