Lecture 1: Trees and Extensions

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1 References

[**TAOCP**] The Art of Computer Programming, D. E. Knuth
[**RLCS**]Introduction to algorithms, T. H. Cormen, C. E. Leiserson, and R. L. Rivest.
[**GTA**]Graph Theoretical Algorithms, Lecture notes, U of Waterloo.
[**AGTCO**]Advance Graph Theory and Combinatorial Optimization.
[**CO**]Combinatorial Optimization,theory and algorithms. B. Korte, J. Vygen, springer,1991
[**CCOM**]A course in Combinatorics. J.H.van Lint, R.M.Wilson.

2 Trees

2.1 Enumeration of trees

1.free tree, orient tree, ordered tree
free tree=unrooted tree
orient tree=rooted tree
order tree=rooted tree + children have orders.

2.Dynamic programming on trees
2.1 Independent set

exercise: given a tree, every nodes has a weight $w(v) \to \mathbb{R}$, find a connected induced subgraph(of course also a tree) of maximum weight.

exercise: given a undirected graph, every nodes has a weight $w(v) \to \mathbb{R}$, find a connected induced subgraph(of course also a tree) of maximum weight. Try to find a polynomial time algorithm or prove it is NP-hard.

3.The matrix tree theorem [TAOCP]pp.378 exercise 18,19

4. graceful tree
See *Towards the Graceful Tree Conjecture: A Survey*

2.2 Relation between Trees and Eular Tours

[TAOCP]pp.372-376,section 2.3.4.2

For a more comprehensive treatness, see *counting spanning trees, Martin Rubey, 2000*

2.3 Minimum Weight Arborescence

In fact, in this subsection, we solve maximum weight brancing problem, which is equivalent to minimum weight arborescence problem.
We use the argument from [CO]pp-122,**Edmonds Branching algorithms**.

See also [AGTCO]pp-5.

3 Extension of Trees

The contents of this section are mainly drawn from [GTA] Lecture 13,14,15,27

3.1 K-trees

3.2 Partial k-trees

3.3 Dynamic Programming on Partial k-trees

3.4 Bounded Treewidth

3.5 K-Outerplanar Has Small Treewidth