

Coloring and Extension

Jian Li

April 12, 2006

Fudan University, Department of Computer Science and Engineering
Shanghai 200433, China

1 References

[MGT] Modern Graph Theory, Béla Bollobás

[AANP] Approximation algorithm for NP-hard problems, edited by D.S.Hochbaum

2 Coloring

chromatic number: $\chi(G)$

[MGT] pp146 Thm 3

Chromatic Polynomial

Planar graph: 4-colorable, Deciding whether a planar graph is 3 colorable is NPC.

Approximation result: $O(n^{\frac{\log \log^2 n}{\log^2 n}})$ -approx A famous semidefinite approach (see ref, Karger, Motwani, Sudan in FOCS94) see also [AANP] chapter 11.

Unapproximability result: $O(n^\delta)$ for some fixed δ , current best $\delta = 1/5$. see [AANP] chapter 10.

Online version:

No algorithm for general graph can be better than $\frac{2n}{\log^2 n}$ -competitive. (In rudolf graduate course)

Lov'asz theta ϑ -function and sandwich theorem. (see ref, Knuth has a good survey) see [AANP] chapter 10.

Chromatic number on random graphs. (see ref)

3 Edge Coloring

edge chromatic number: $\chi'(G)$

Vizing's Theorem : $\Delta \leq \chi'(G) \leq \Delta + 1$ [AGTCO] pp12, [MGT] pp152 thm3.1

Deciding whether $\chi'(G) = \Delta$ or $\chi'(G) = \Delta + 1$ is NPC, [AGTCO] pp12

4 List Coloring

list chromatic number: $\chi_l(G)$

list edge chromatic number: $\chi'_l(G)$

[MGT]pp161, thm12(For planar graph G , $\chi_l(G) \leq 5$),

thm13, thm14(Galvin's theorem, see also [CO]pp194):For bipartite graph G , $\chi'_l(G) = \chi'(G)$