

Matching and Extensions

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1 matching

1. matching in bipartite graph

given bipartite graph $G(U, V; E)$, where

$\min(\text{edge cover}) = \max(\text{independent set}) = |U| + |V| - \max(\text{matching})$ (König's edge cover theorem)

$\max(\text{matching}) = \min(\text{vertex cover})$ (König's matching theorem)

application: CCOM pp48 thm 5.4

(see AGTCO pp7-8, CO pp206)

Hall's marriage theorem (important) (CLRS pp669-exe26.3-4, AGTCO pp9, exe2.3, 2.4, CO pp206, CCOM pp43)

perfect matching. SDR (System of Distinct Representative).

applications: CLRS pp669-exe26.3.5. CCOM pp48 thm5.5, pp186 thm17.1.

2. matching in general graph

Edmonds' Algorithm (CO pp219) $O(n^3)$ time (1965)

best known algorithm so far: $O(\sqrt{nm})$ time, by Micali and Vazirani (FOCS 1980)

3. Tutte Matrix (CO pp208)

here we mention a closely related result: $\text{Perm}(B(G)) = \#(\text{perfect matching in } G)$, G is a bipartite graph, $B(G)$ is bipartite adjacency matrix

4. weighted matching

Edmonds' algorithm ($O(n^3)$ time implementation due to Gabow)

5. application of perfect matching

(1) (odd and even shortest path) CO pp259 exe12

(2) cycle cover

(3) see (2-matching)

(4) see (path matching)

2 2-matching and b-matching

2-matching (CO pp261)

1.reduce 2-matching to perfect matching problem see [combinatorial optimization](#); A.Schrijver PP526

2.mention: 3-cycle free 2-matching (see [PHDThesis]Havtvigsen,CMU), 4-cycle free 2-matching is NPC.

3.Application: 4/3 approximation for (1,2)-TSP

b-matching(CO pp261)

3 path matching

see.Length-Constrained Path-Matchings in Graphs

see William H. Cunningham. Matching, matroids, and extensions

4 T-join and T-cut

T-join(CO pp265)is generalization of

1.Chinese Postman Problem

2.Eularian subgraph

3.S-T shortest path

4.perfect matching problem

T-Cut, (CO pp269)

Application: CO-pp276,exe 6,7

1.odd cycle cover

2.Max-cut in planar graph(in P).(note it is NPC in general graph)