CMSC 311
Solutions to Homework 6

Problem 1.

(a)

a_0	a_1	a_2	a_3	b_1	b_0
0	0	0	1	1	1
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	0	0	1

(b)

$$b_0 = \overline{a_0}\,\overline{a_1}\,\overline{a_2}\,a_3 \vee \overline{a_0}\,a_1\,\overline{a_2}\,\overline{a_3} \vee a_0\,a_1\,\overline{a_2}\,a_3 \vee a_0\,a_1\,a_2\,\overline{a_3}$$

$$b_1 = \overline{a_0}\,\overline{a_1}\,\overline{a_2}\,a_3 \vee \overline{a_0}\,\overline{a_1}\,a_2\,\overline{a_3} \vee \overline{a_0}\,a_1\,a_2\,a_3 \vee a_0\,\overline{a_1}\,a_2\,a_3$$

(c) OMITTED

(d)

$$b_0 = a_0\,a_1 \vee \overline{a_0}\,\overline{a_2} = a_1\,\overline{a_3} \vee \overline{a_2}\,a_3$$

$$b_1 = \overline{a_0}\,\overline{a_1} \vee a_2\,a_3 = \overline{a_0}\,a_3 \vee \overline{a_1}\,a_2$$

Problem 2.

(a) For $a \wedge b$ have a use top input, b use bottom input, and use a to select.

(b) For $a \vee b$ have a use top input, b use bottom input, and use b to select.

(c) For $\overline{a}$ have 1 use top input, 0 use bottom input, and use a to select.

(d) 2×1 multiplexers with 0 and 1 inputs are universal in the sense they can emulate any Boolean gate.

Problem 3.

(a) Let $n = 2^k$. If we allowed arbitrarily large fan-in for the gates, there would be n AND gates and one OR gate. Each AND gate would have k select lines and one data line for a total of $k+1$ lines. The OR gate would have n input lines, one for each data item. An AND or OR gate with $m > 2$ inputs can be emulated with $m-1$ two-input gates. So the total number of gates is:

$$nk + n - 1 = n \lg n + n - 1$$

(b) The number of levels of gates to emulate an AND or OR gate with $m > 2$ inputs is $\lceil \lg m \rceil$ using a ladder tournament arrangement. So each AND gate needs $\lceil \lg(k+1) \rceil$ levels and the OR gate needs $\lceil \lg n \rceil = k$ levels. The total number of levels of gate delay is

$$k \ + \ \lceil \lg(k+1) \rceil \quad = \quad \lg n \ + \ \lceil \lg(\lg n + 1) \rceil$$

Problem 3 (alternative solution). Let $n = 2^k$.

We can construct an $n \times 1$ multiplexer recursively by taking putting two $n/2 \times 1$ multiplexers side-by-side, which takes care of all n input lines, and selecting using the same $k-1$ select lines for each multiplexer. The two outputs are input into a 2×1 multiplexer and the last select line is used to select between them.

A 2×1 multiplexer uses three gates: two AND gates and one OR gate. It has two levels of gate delay.

(a) The size $S(n)$ of an $n \times 1$ multiplexer satisfies the recurrence:

$$S(n) \quad = \quad 2S(n/2) + S(1) \quad = \quad 2S(n/2) + 3$$

where $S(1) = 3$. The solution to this recurrence is $3n - 3$.

(b) The gate delay $D(n)$ of an $n \times 1$ multiplexer satisfies the recurrence:

$$D(n) \quad = \quad D(n/2) + D(1) \quad = \quad D(n/2) + 2$$

where $D(1) = 2$. The solution to this recurrence is $2 \lg n$.

Problem 3 (NOTE). Neither solution takes into account the cost of broadcasting the select lines to as many as $n/2$ AND gates.