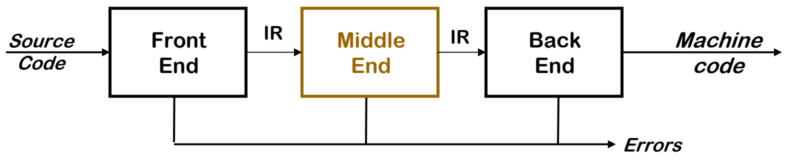
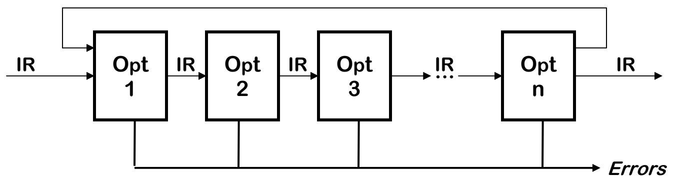
Traditional Three-pass Compiler



Code Improvement (or Optimization)

- Analyzes IR and rewrites (or transforms) IR
- Primary goal is to reduce running time of the compiled code
  - May also improve space, power consumption, …
- Must preserve "meaning" of the code
  - Measured by values of named variables
  - A course (or two) unto itself

The Optimizer (or Middle End)



Modern optimizers are structured as a series of passes

Typical Transformations

- Discover & propagate some constant value
- Move a computation to a less frequently executed place
- Specialize some computation based on context
- Discover a redundant computation & remove it
- Remove useless or unreachable code
- Encode an idiom in some particularly efficient form

Why are optimizers needed?

Reduce programmer effort

- automatically generate efficient code
- less work for programmer
- below “optimal” hand-optimized code

Undo high-level abstractions

- some optimizations not possible for language
- flatten control flow to branches
- convert method lookups to subroutine calls
- map data structures to addresses

Maintain performance portability

- performance depends on architecture
- optimizations by programmer too specific
- compiler can customize program for processor

Code optimizations

Reduce execution time

- historically, to avoid assembly coding
- support higher levels of abstraction
- support more complex processors
- important applications: science, databases

Reduce space

- historically, small expensive memories may trade space for speed
- space may reduce speed (caches)
- new areas: internet applets, embedded processors

Level of optimization

- source code
- intermediate representation
- binary machine code
- at run-time

Code optimization

How can optimizations improve code quality?

Machine-independent transformations

- remove unnecessary computations
- simplify control structures
- move code to a less frequently executed place
- specialize some general purpose code
- find useless code and remove it
- expose opportunities (enable) for other optimizations

Machine-dependent transformations

- replace complex operation with simpler one
- exploit special instructions
- exploit memory hierarchy (registers, cache)

Types of optimization

- classical - reduce the number/cost of instructions execute
- register allocation - keep values in registers, eliminate loads/stores
- instruction scheduling - hide instruction latency, exploit instruction-level parallelism
- data locality - keep data accesses in faster levels of memory hierarchy (registers, cache, TLB, memory)
- Multiprocessing - compute in parallel on multiple processors

Optimization framework

- ideally, maintain separation of concerns
- in practice, integrate optimization algorithms

Code optimization

Three considerations arise in applying a transformation.

- Safety - Does applying the transformation change the results of executing the code?
- Profitability - Is there a reasonable expectation that applying the transformation will improve the code?
- Opportunity - Can we efficiently and frequently find places to apply optimization?

Need a clear understanding of these issues.

Profitability is particularly tricky...

Learn how the compiler decides when transformations will be applicable, safe, and profitable.

Classical transformation examples

- Assumption: Anything executed once takes no time. Gains achieved by looping at program loops.
- Break program into straightline code segments, bounded by branches (Basic Blocks)
- Connect basic blocks together into a program graph
- Find innermost loop
- Find loop invariant expressions and move them to head of loop
- Find common subexpressions and combine them
- Move loop invariant expressions out of loop
- Repeat process with next outer loop

Classical Transformation Examples

- Unreachable code - eliminate code not reached during program execution

```
        goto L:
        unreachable code
  L:                              (Delete all this code)
```

- Control-flow simplification - remove jumps to jumps by analyzing targets of jumps

```
        goto L                    (goto M)
        code
  L:    goto M
        code
  M:
```

- Algebraic simplification - simplify arithmetic expressions by analyzing expression trees

```
      A := 0
      C := B + A                  (C:= B)
```

Classical Transformation Examples

- Constant folding - replace constant expressions with result

```
     A := 5
     B := 6
     C := B + A             (C:=11)
```

- Idiom recognition - replace operations with less expensive idioms

```
     B := A * 16            (Shift right)
     D := B / 4
```

- Available expressions - reuse values always available

```
     C := B + A
     D := B + A             (D:=C)
```

- Dead code elimination - eliminate unnecessary computations

```
     A := 5                 (Delete this statement)
     A := 6
```

- Copy propagation - propagate names into copy instructions

```
     B := A                 (Delete this statement)
     C := B                 (C:=A)
```

Classical Transformation Examples

- Procedure integration
  call S(A,B) *(expand as inline code)*
- Loop unrolling

```
for I:= 1 to 100;
A[I] := A[I] + B[I];
end;
```

becomes

```
for I:= 1 to 100 by 2;
   A[I] := A[I] + B[I];
   A[I+1] := A[I+1] + B[I+1];
end;
```

increases opportunity for parallelism (multiple processors or pipelining) but increases program size

Classical Transformation Examples

- Jamming -- opposite of loop unrolling

```
for I:= 1 to 100;
   A[I] := A[I] + B[I];
   end;
for I:= 1 to 100;
   C[I] := C[I] + D[I];
   end;
   becomes
for I:= 1 to 100;
   A[I] := A[I] + B[I];
   C[I] := C[I] + D[I];
   end;
```

increases opportunity for parallelism (multiple processors or pipelining)

- Common subexpression elimination

```
A:= B+C;
D:= B+C;
```

becomes

```
A:= B+C;
D:= A;
```

reduces size of program, reduces execution time and can increase execution time of program (How?)

Classical Transformation Examples

- Code motion - reduces execution of redundant instructions

```
    while X<Y do
       A := B+C;
       X:= X+1;
       end;
becomes
    A := B+C;
    while X<Y do
       X:= X+1;
    end;
```

Basic blocks -- Fundamental concept for all code improvement algorithms --
sequence of code where control enters at top, exits at bottom,
no branch/halt except at end

- Construction algorithm (for 3-address code)
  determine set of leaders
  first statement
  target of goto or conditional goto
  statement following goto or conditional goto
- add to basic block all statements following leader up to next leader or end of program

```
    A := 0                 (Block 1)
    if (<cond>) goto L
    A := 1                 (Block 2)
    B := 1                 (Block 2)
L:   C := A                (Block 3)
```

Scope of optimizations

Scope

- peephole --- across a few instructions
- local --- within basic block
- global --- across basic blocks
- interprocedural --- across procedures

Some optimizations may be applied locally or globally (e.g., dead code elimination):

```
A := 0      A := 0
A := 1       if (<cond>) goto L
B := A       A := 1
             B := A
```

Some optimizations require global analysis (e.g., loop-invariant code motion):

```
while (<cond>) do
   A := B + C
   foo(A)
end
```

The Role of the Optimizer

- The compiler can implement a procedure in many ways
- The optimizer tries to find an implementation that is "better"
  - Speed, code size, data space, …

To accomplish this, it

- Analyzes the code to derive knowledge about run-time behavior
  - Data-flow analysis, pointer disambiguation, …
  - General term is "static analysis"
- Uses that knowledge in an attempt to improve the code
  - Literally hundreds of transformations have been proposed
  - Large amount of overlap between them

Nothing "optimal" about optimization

- Proofs of optimality assume restrictive & unrealistic conditions

General optimization process

- Generate graph of program, based on basic blocks
  1. Compute live/dead analysis for all variables
  2. Redundancy
- Look for common subexpressions using liveness analysis to determine if variable values have been changed to see if two expressions have the same value

1. Live/dead variable analysis

- Determine path through a program where a variable's value does not change
- If a and b do not change, then the expression (a+b) anywhere along this path contains the same value
  - Can compute (a+b) once and use computed value for each occurrence.
- If two variables (e.g., temporaries) are not used along the same path, they can share the same memory location or register
  - Better use of registers
  - Less storage to use

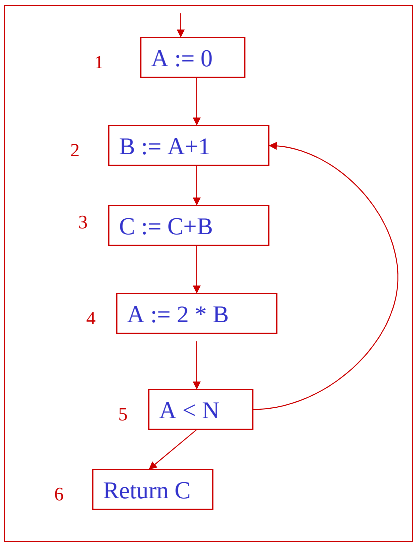
1. Definitions

- Live – a variable is live if its value will be used before the variable is redefined
- Def – The def of a variable is the set of graph nodes that define a value to that variable
- Use – The use of a variable is the set of graph nodes that access the value of a variable

- Def and Use vectors are a syntactic property of a program.
- Give the graph structure and the Def and Use vectors at each node of a program graph, you can compute the liveness of each variable.

1. Example liveness property

- B is live 2→4
- C is live on entry,
  - Live 1→ 3,
  - Live 3 → 3,
  - Live 3→ 6
- A is live 1 → 2
  - Live 4 →2
- Example:
  - Def(3) = {C}
  - Use(3) = {B, C}
  - Live(3) = {B, C}



1. Computing liveness

- Live-in(n) – Variables live as input to block n
- Live-out(n) – Variables live as output from block n
- in(n) = use(n) $\cup$ (out(n) – def(n))
- out(n) = $\cup$ in(s) for s $\in$ succ(n)

Solving this set of equations for all n gives the liveness property for each variable (e.g., see page 225 of text).
  - Time to compute is: O(N^4) as worst case
  - Usual time is O(N) to O(N^2)

2. Redundancy Elimination as an Example

An expression x+y is redundant if and only if, along every path from the procedure's entry, it has been evaluated, and its constituent subexpressions (x & y) have <u>not</u> been re-defined.

If the compiler can prove that an expression is redundant

- It can preserve the results of earlier evaluations
- It can replace the current evaluation with a reference

Two pieces to the problem

- Proving that x+y is redundant
- Rewriting the code to eliminate the redundant evaluation

One technique for accomplishing both is called <u>value numbering</u>

2. Data Flow Equations for Availability

- Constants
  - > DEF(b) — subexpressions defined in b and available on exit
  - > NOTKILLED(b) — subexpressions that are not killed in b
    - → A subexpression is killed if either input is assigned to
- Computing AVAIL(b) — the set of expressions available on input to block b

$$\text{AVAIL}(b) = \cap_{p \text{ in } \text{Pred}(b)}(\text{DEF}(p) \cup (\text{AVAIL}(p) \cap \text{NOTKILLED}(p)))$$

- What is the starting value for AVAIL(b)?
- What is the problem with this formulation?

2. Value Numbering *A 1960's Idea*

The key notion (Balke 1968 or Ershov 1954)

- Assign an identifying number, V(n), to each expression
  - > V(x+y) = V(j) iff x+y and j have the same value ∀ inputs
  - > Use hashing over value numbers to make it efficient
- Use these numbers to improve the code

Improving the code

- Replace redundant expressions
- Simplify algebraic identities
- Discover constant-valued expressions, fold & propagate them

- This technique was invented for low-level, linear IRs
- Equivalent methods exist for trees (build a DAG)

2. Local Value Numbering

The algorithm

For each operation *o* = <operator, o_1, o_2> in the block

1 Get value numbers for operands from hash lookup
2 Hash <operator,VN(o_1),VN(o_2)> to get a value number for *o*
3 If *o* already had a value number, replace *o* with a reference
4 If o_1 & o_2 are constant, evaluate it & replace with a loadI

If hashing behaves, the algorithm runs in linear time

- > If not, try multi-set discrimination

Handling algebraic identities

- Case statement on operator type
- Handle special cases within each operator

2. Local Value Numbering

An example

Original Code	**With VNs**	**Rewritten**
$a \leftarrow x + y$	$a^3 \leftarrow x^1 + y^2$	$a^3 \leftarrow x^1 + y^2$
$* \; b \leftarrow x + y$	$* \; b^3 \leftarrow x^1 + y^2$	$* \; b^3 \leftarrow a^3$
$a \leftarrow 17$	$a^4 \leftarrow 17$	$a^4 \leftarrow 17$
$* \; c \leftarrow x + y$	$* \; c^3 \leftarrow x^1 + y^2$	$* \; c^3 \leftarrow a^3$ (oops!)

Two redundancies:
- **Eliminate stmts with a $*$**
- **Coalesce results ?**

Options:
- **Use $c^3 \leftarrow b^3$**
- **Save a^3 in t^3**
- **Rename around it**

2. Local Value Numbering

Example (continued)

Original Code	**With VNs**	**Rewritten**
$a_0 \leftarrow x_0 + y_0$	$a_0^{\;3} \leftarrow x_0^{\;1} + y_0^{\;2}$	$a_0^{\;3} \leftarrow x_0^{\;1} + y_0^{\;2}$
$* \; b_0 \leftarrow x_0 + y_0$	$* \; b_0^{\;3} \leftarrow x_0^{\;1} + y_0^{\;2}$	$* \; b_0^{\;3} \leftarrow a_0^{\;3}$
$a_1 \leftarrow 17$	$a_1^{\;4} \leftarrow 17$	$a_1^{\;4} \leftarrow 17$
$* \; c_0 \leftarrow x_0 + y_0$	$* \; c_0^{\;3} \leftarrow x_0^{\;1} + y_0^{\;2}$	$* \; c_0^{\;3} \leftarrow a_0^{\;3}$

Renaming:
- **Give each value a unique name**
- **Makes it clear**

Notation:
- **While complex, the meaning is clear**

Result:
- **$a_0^{\;3}$ is available**
- **rewriting just works**

Simple Extensions to Value Numbering

Constant folding

- Add a bit that records when a value is constant
- Evaluate constant values at compile-time
- Replace with load immediate or immediate operand
- No stronger local algorithm

Algebraic identities

- Must check (many) special cases
- Replace result with input VN
- Build a decision tree on operation

Identities: (Click)
$x \leftarrow y$, $x+0$, $x-0$, $x*1$, $x \div 1$, $x-x$, $x*0$, $x \div x$, $x \vee 0$, $x \wedge$ 0xFF…FF, max(x,MAXINT), min(x,MININT), max(x,x), min(y,y), and so on ...

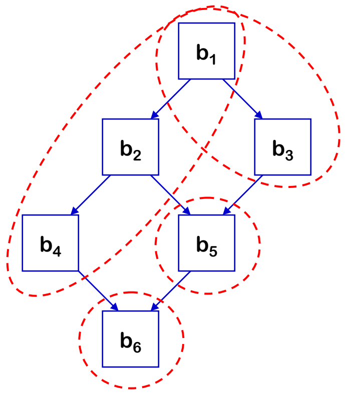
With values, not names

Handling Larger Scopes

Extended Basic Blocks

- Initialize table for b_i with table from b_{i-1}
- With single-assignment naming, can use scoped hash table

Otherwise, it is complex



<u>The Plan:</u>

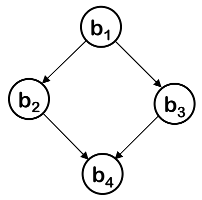
→ Process b_1, b_2, b_4
Pop two levels
→ Process b_3 relative to b_1
→ Start clean with b_5
→ Start clean with b_6

Using a scoped table makes doing the full tree of EBBs that share a common header efficient.

Handling Larger Scopes

To go further, we must deal with merge points

- Our simple naming scheme falls apart in b_4
- We need more powerful analysis tools
- Naming scheme becomes SSA



$$\begin{tikzcd} & b_1 \arrow[dl] \arrow[dr] & \\ b_2 \arrow[dr] & & b_3 \arrow[dl] \\ & b_4 & \end{tikzcd}$$

This requires global data-flow analysis

"Compile-time reasoning about the run-time flow of values"

1 Build a model of control-flow
2 Pose questions as sets of simultaneous equations
3 Solve the equations
4 Use solution to transform the code

Examples: LIVE, REACHES, AVAIL

Constant Propagation

- Goal: Produce an algorithm that will propagate all constants in a procedure, replacing constant expressions with the result of evaluating the expression at compile time
- Strategy:
  - > Construct a graph that maps definitions to uses within a procedure — def-use chains
  - > Propagate constants forward from points of constant definitions along def-use chains
  - > Evaluate new constant expressions whenever they are identified
  - > Stop when no more constants are available
- Challenges
  - > Constructing def-use chains
  - > Identifying constant expressions

Constructing Def-Use Chains

- Perform REACHES calculation

$$\text{REACHES}(b) = \cup_{p \in pred(b)} \text{REACHESOUT}(p)$$

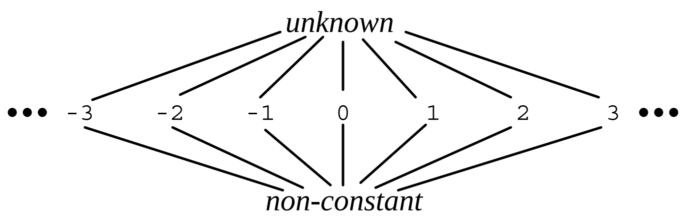
$$\text{REACHESOUT}(b) = \text{DEFSOUT}(b) \cup (\text{REACHES}(b) \cap \text{NOREDEF}(b))$$

- At each use of variable x, construct a DEF-USE chain to x from each definition y in REACHES at x
- Note: REACHES sets easy to propagate forward in basic blocks

Dead Code Elimination

```
worklist := {absolutely useful statements};
while worklist ≠ ∅ do begin
    x := an arbitrary element of worklist;
    worklist := worklist – {x};
    mark x useful;
    for all (y,x) ∈ DefUse do
        if y is not marked useful
        then worklist := worklist ∪ {y};
end
delete every statement that is not marked useful;
```

Constant Propagation Lattice

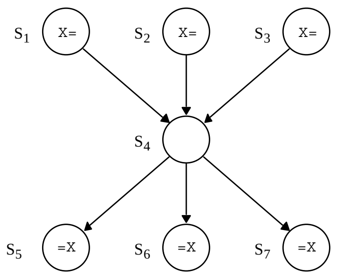


Constant Propagation Algorithm

```
for all statements s in the program do begin
    for each output v of s do valout(v,s) := unknown;
    for each input w of s do
        if w is a variable then valin(w,s) := unknown;
        else valin(w,s) := the constant value of w;
end
worklist := {all statements of constant form, e.g., X = 5};
while worklist ≠ ∅ do begin
    choose and remove an arbitrary statement x from worklist;
    let v denote the output variable for x;
    newval := m(x)(valin(v,x), for all inputs v to x);
    if newval ≠ valout(v,x) then begin
        valout(v,x) := newval;
        for all (x,y) ∈ DefUse do begin
            oldval := valin(v,y);
            valin(v,y) := oldval ⊓ valout(v,x);
            if valin(v,y) ≠ oldval then worklist := worklist ∪ {y};
        end
    end
end
```

Advantages and Disadvantages

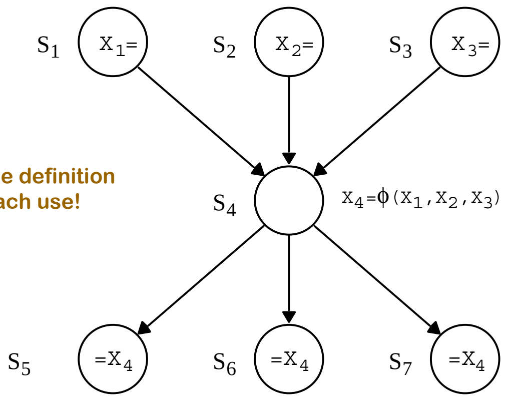
- Advantages
  - Linear in the size of the Def-Use graph
    - → Why?
- Disadvantage
  - Def-Use graph could be large



Shrinking the Graph: SSA

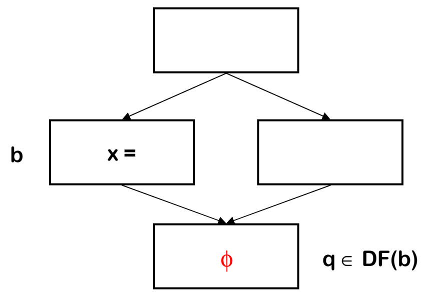
- Static Single-Assignment Form

At most one definition reaches each use!



Constructing SSA

- Find points of insertion for ϕ-functions and insert them
  - Where should they go?



Put a ϕ-function for x in every block in the dominance frontier for block b

The dominance frontier DF(b) for a given block b is the set of blocks q such that some predecessor of q is dominated in the control-flow graph by b, but q itself is not strictly dominated by b.

Dominators

- A node x in directed graph G with a single exit node predominates (or dominates) node y in G if any path from the entry node of G to y must pass through x.
- The immediate dominator of a block x is the block y in dominators(x) such that dominators(y) = dominators(x) - {x}
- How do we compute dominators(b)?

$$\text{DOMINATORS}(b) = \{b\} \cup \bigcap_{p \text{ in preds}(b)} \text{DOMINATORS}(p)$$

- Is it really this easy?

Computing Dominance Frontiers

Find the immediate dominator relation idom for the control-flow graph G; (For a control-flow graph with a single entry, this relation forms a tree, with the entry node as the root.)

Let l be a topological listing of the dominator tree such that, if x dominates y, then x comes after y in l;

```
while $l \neq \emptyset$ do begin
    let x be the first element of l;
    remove x from l;

    for all control flow successors y of x do
        if $idom(y) \neq x$ then $DF(x) = DF(x) \cup \{y\}$;

    for all z such that $idom(z) = x$ do
        for all $y \in DF(z)$ do
            if $idom(y) \neq x$ then $DF(x) = DF(x) \cup \{y\}$;
end
```

Algorithms on SSA

- Dead code elimination and constant propagation work unchanged, assuming a meaning is constructed for ϕ-functions;
  - The edge set should be much smaller, so the algorithms should run faster
- Many other algorithms can exploit the single-assignment property
  - What about value numbering?
  - Since each value has a unique name, you can do value number on SSA in complex control flow