Answer all questions in the exam book. If you do not understand something, write down what you think you are solving and answer that.

1 [20] (a) Show \( \{ a^{3n} \mid n \geq 0 \} \) is a regular language.
   (b) Show \( \{ a^{2n} \mid n \geq 0 \} \) is a context free language.
   (c) Show \( \{ a^k \mid k = n^3 \text{ and } n \geq 0 \} \) is not a context free language.

2 [10]. Consider the grammar \( G = (N, T, P, S) \) with \( P \) being the rules:
   \[
   S \rightarrow c \mid aSXB
   X \rightarrow dS \mid \lambda
   \]
   Determine whether \( G \) is LL(1). If so, give the parse tables; if not, why not?

3 [15]. Give the deterministic finite state automaton that describes the same set:
   (a) \( 01^*(01)^*(01) \)
   (b) \( G = (N, T, P, S) \) with \( P \) being: \( S \rightarrow SS \mid a \)

4 [10] For each of the following, give a regular grammar that describes the same set:
   (a) \( 0(1)0(1)01^* \)

5 [20] Let \( G \) be a context free grammar with the following productions and consider the string generated by the grammar: \( abbabbabc \).
   \[
   S \rightarrow aXS \mid Y
   X \rightarrow bX \mid b
   Y \rightarrow cY \mid c
   \]
   (a) What are all the simple phrases in the string?
   (b) What is the handle of the string?
   (c) Give a parse tree for the string.
   (d) Give the leftist derivation for the string.

6 [5] (a) If an entry in the LL(1) parsing table has more than one applicable production, then the following must be true (List all true answers):
   (1) The grammar is not LL(1).
   (2) The language generated by the grammar is not LL(1).
   (3) The grammar is ambiguous.
   (b) If you arbitrarily pick one of these multiple entries for that table position to create an LL(1) table, then the following may be true (List all true answers):
   (1) The table will parse all strings in the language.
   (2) The table will parse some strings in the language.
   (3) The table will parse no strings in the language.

7 [10] A context free grammar is left recursive if there is some rule of the form \( A \rightarrow aA \) for some nonterminal \( A \). Show that for any context free grammar that has left recursive rules, there is another context free grammar that does not have left recursive that generates the same set of strings.

8 [10] Let \( R \) be a regular set. Define \( S \) as follows:
   \[
   S = \{ w \mid \text{there is some string } x \text{ such that } \text{length}(x) = \text{length}(w) \text{ and } wx \in R \}
   \]
   (i.e., \( S \) is the first half of strings that are in \( R \).) Is \( S \) regular? Prove your answer. [ This is hard and few are expected to solve it. Finish rest of text before spending time on it. ]