Prove that for any $n$, the language $L = \{a^n b^n \mid n \geq 0\}$ is not context-free.

**Proof**

Assume for contradiction that $L$ is context-free. Then there exists a context-free grammar for $L$.

Let $G = (V, \Sigma, R, S)$ be the context-free grammar that generates $L$.

Consider the string $a^k b^k$ for some $k > 0$.

Then $a^k b^k \in L$, and there exists a derivation in $G$:

$$S \Rightarrow^* a^{k-1} \Rightarrow^* a^{k-1} b \Rightarrow^* a^{k-1} b b \Rightarrow^* a^{k} b^k$$

But $a^{k} b^k$ has more $b$'s than $a$'s, contradicting the grammar's generation.

Therefore, $L$ cannot be context-free.

**Note:** The Pumping Lemma can be used to show something is not context-free, not to show that something is.