

**Sample Axiomatic problems**  
**September 17, 2003**

1. (a) What is the precondition that completes the specification of the following program?

(b) Prove the following program using Hoare axioms:

```
{precondition}
while x>b do
    x = x-1
end
y = x+a;
{y = a+b}
```

2. Give Hoare-like axioms to the following constructs:

(a)  $x++$             Add 1 to x

(b)  $x += y$         Add y to x

(c)  $x \leftrightarrow y$        $x = y'$  and  $y = x'$ , where  $char'$  is the original value of char. (i.e., switch values)

3. Prove correctness of the following:

```
{true}
s = 0;
i = 0;
while i < 3 do
    s += a;
    a ↔ b;
    s += a;
    i++;
end
{ s=3*(a+b) }
```

4. What is the precondition and prove the correctness of the following:

```
{precondition}
i = 0;
while i<n do
    i = i+1;
    A[i] = B[i]
end
{ A = B' } (B' = original value of B)
```

5. Consider the following axioms:

(1)  $f(g,x) = x$

(2)  $f(h(x),y) = h(f(x,y))$

Prove:  $f(h(x),y) = f(x,h(y))$

6. Given the axioms for addition:

(1)  $add(x,0) = x$

(2)  $add(x,succ(y)) = succ(add(x,y))$

(a) Give axioms for  $even(x)$  meaning x is an even value

(b) prove  $even(x) \wedge even(y) \rightarrow even(add(x,y))$

(c) Define  $odd(x)$  similarly, and prove:  $odd(x) \wedge odd(y) \rightarrow even(add(x,y))$