Sample Axiomatic problems
September 17, 2003

1. (a) What is the precondition that completes the specification of the following program?
   (b) Prove the following program using Hoare axioms:

   
   \begin{verbatim}
   {precondition}
   while x>b do
       x = x-1
   end
   y = x+a;
   {y = a+b}
   \end{verbatim}

2. Give Hoare-like axioms to the following constructs:
   (a) x++ Add 1 to x
   (b) x += y Add y to x
   (c) x ↔ y x = y’ and y = x’, where char’ is the original value of char. (i.e.,
switch values)

3. Prove correctness of the following:

   \begin{verbatim}
   {true}
   s = 0;
   i = 0;
   while i < 3 do
       s += a;
       a ↔ b;
       s += a;
       i++
   end
   { s=3*(a+b) }
   \end{verbatim}

4. What is the precondition and prove the correctness of the following:

   \begin{verbatim}
   {precondition}
   i = 0;
   while i<n do
       i = i+1;
       A[i] = B[i]
   end
   {A = B’ } (B’ = original value of B)
   \end{verbatim}

5. Consider the following axioms:
   (1) f(g,x) = x
   (2) f(h(x),y) = h(f(x,y))

   Prove: f(h(x),y) = f(x,h(y))

6. Given the axioms for addition:
   (1) add(x,0) = x
   (2) add(x,succ(y)) = succ(add(x,y))

   (a) Give axioms for even(x) meaning x is an even value
   (b) prove even(x) ∧ even(y) \rightarrow even(add(x,y))
   (c) Define odd(x) similarly, and prove: odd(x) ∧ odd(y) \rightarrow even(add(x,y))