A New Min-Max Regret Robust Optimization Approach for Solving Two-Stages Mixed Integer Linear Optimization Problems with Full Factorial Scenario Design

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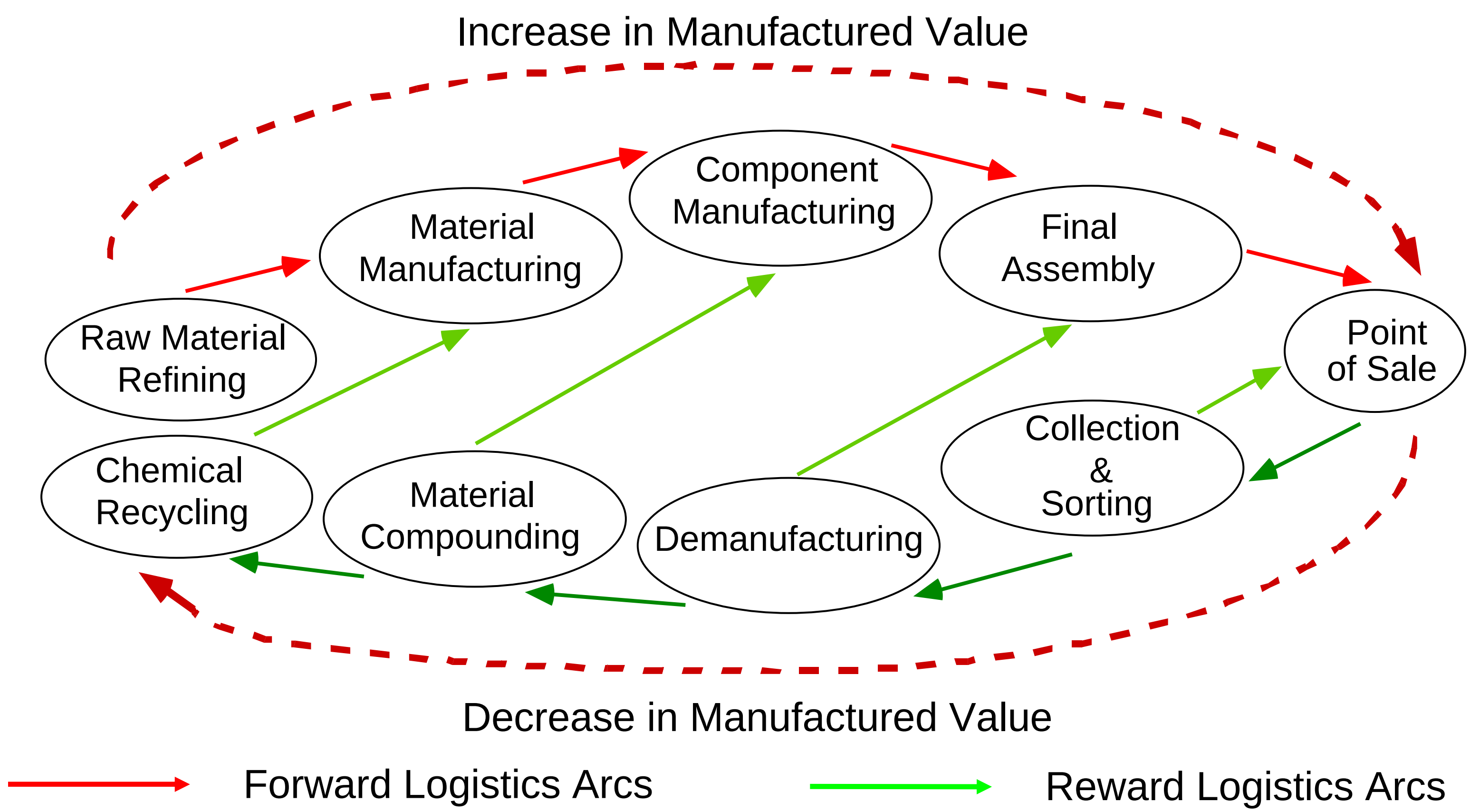
Workshop on Decision Making in Adversarial Domains

May 23rd, 2005

Decision Making using MILP

Example: Design of Supply Chain Systems



Decision Making using MILP

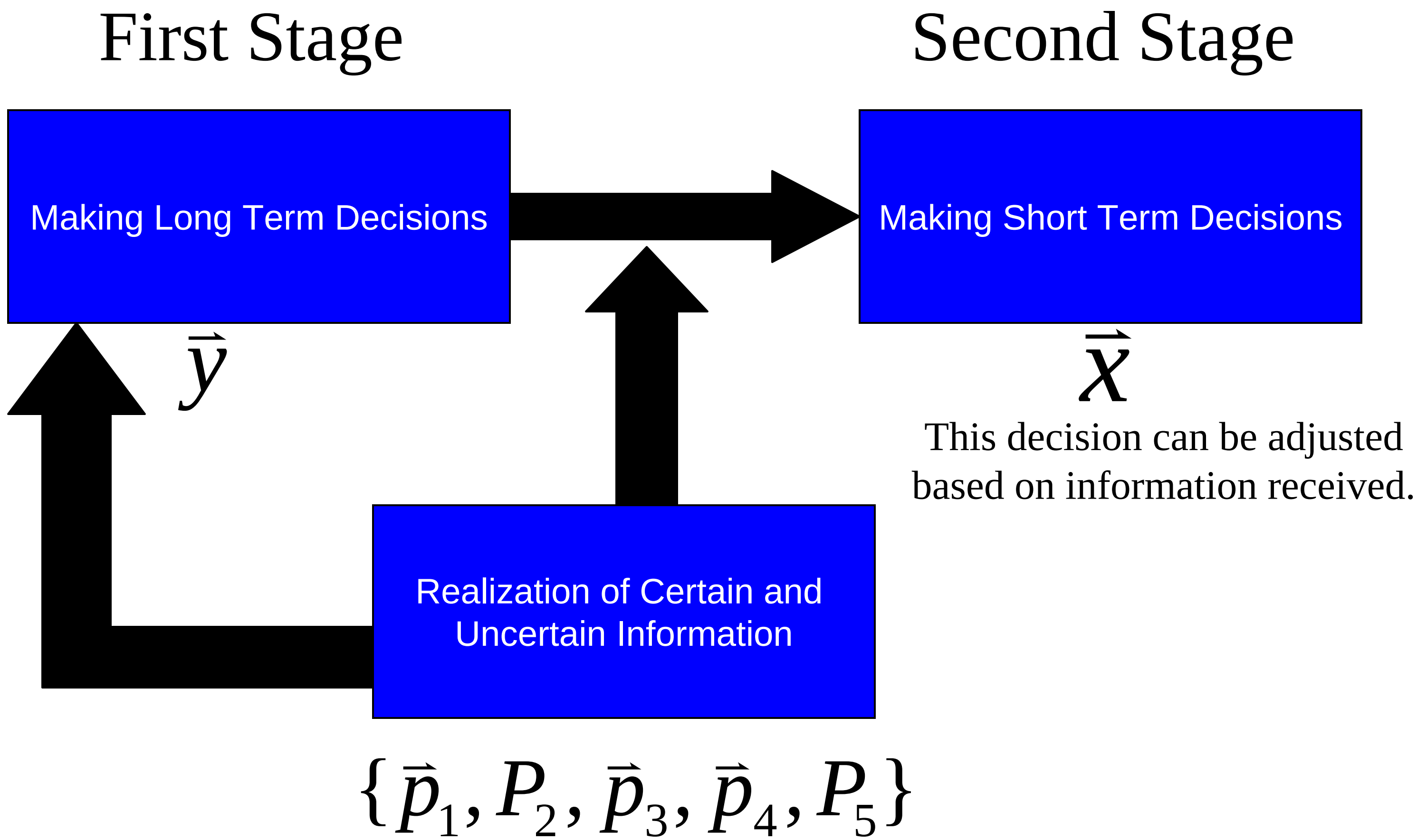
$$\max_{\vec{x}\ ,\vec{y}} \vec{p}_4^{\,T}\vec{x} + \vec{p}_1^{\,T}\vec{y}$$

$$s.t. \quad P_5\vec{x} \le \vec{p}_3 + P_2\vec{y}$$

$$\vec{x} \ge 0 \text{ and } \vec{y} \in \{0,1\}^{|\vec{y}|}$$

(MILP1 Model)

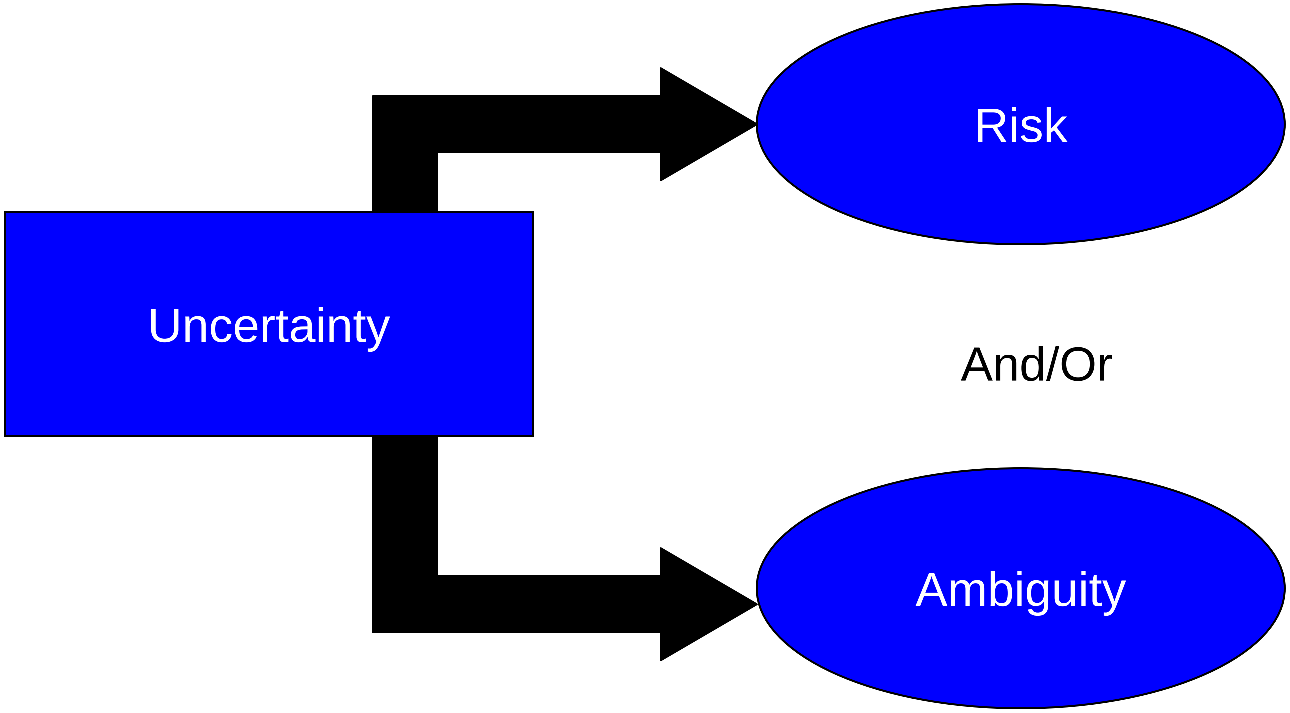
Two-Stages Decision Making



Decision Making Under Uncertainty

How can we make decision when there exists uncertainty in the problem?

There are two important types of uncertainty.



Parameters are random variables with enough historical data.

Parameters are quite unknown with no or few historical data.

Decision Making Under Ambiguity

The common objectives are:

- Finding the solution with the best worst case over all possible scenarios. (Absolute Robust Definition)

- Finding the solution that minimize the maximum regret from optimal objective function under perfect information over all possible future scenarios. (Deviation Robust Definition)

- Finding the solution that minimize the maximum relative regret from optimal objective function under perfect information over all possible scenarios. (Relative Robust Definition)

Deviation Robust Decisions

The major focus of this research is to determine the discrete decisions that performs well across all possible input scenarios.

(This discrete solution will be defined as the robust solution)

Let Ω represents the set of all possible future scenarios of the problem.

Let O_ω^* represents the optimal objective function value if the perfect information is given under scenario $\omega \in \Omega$.

Let R_ω represents the objective function value for the robust solution under scenario $\omega \in \Omega$.

Robust Measure : Minimize the maximum deviation from optimal

$$\min_{x_\omega, y}(\max_{\omega}(O_\omega^* - R_\omega))$$

or

$$\text{minimize } \delta$$
$$\delta \geq O_\omega^* - R_\omega \text{ for all } \omega$$

Extensive Form Model for Two-Stages Deviation Robust Optimization Problem

$$\min_{\delta, y, x_\omega} \quad \delta$$

$$s.t.$$

$$\left.\begin{aligned} &\delta \geq O_\omega^* - R_\omega \\ &R_\omega = \vec{p}_{4\omega}^T \vec{x}_\omega + \vec{p}_{1\omega}^T \vec{y} \\ &P_{5\omega} \vec{x}_\omega \leq \vec{p}_{3\omega} + P_{2\omega} \vec{y} \\ &\vec{x}_\omega \geq \vec{0} \end{aligned}\right\} \quad \forall \omega \in \Omega$$

$$\vec{y} \in \{0,\ 1\}^{|\vec{y}|}$$

(MILP2 Model)

Where

$$O_\omega^* = \left\{\begin{aligned} &\max_{x_\omega, y} \vec{p}_{4\omega}^T \vec{x}_\omega + \vec{p}_{1\omega}^T \vec{y} \\ &s.t. \quad P_{5\omega} \vec{x}_\omega \leq \vec{p}_{3\omega} + P_{2\omega} \vec{y} \\ &\qquad \vec{x}_\omega \geq \vec{0} \text{ and } \vec{y} \in \{0,\ 1\}^{|\vec{y}|} \end{aligned}\right\} \quad \forall \omega \in \Omega$$

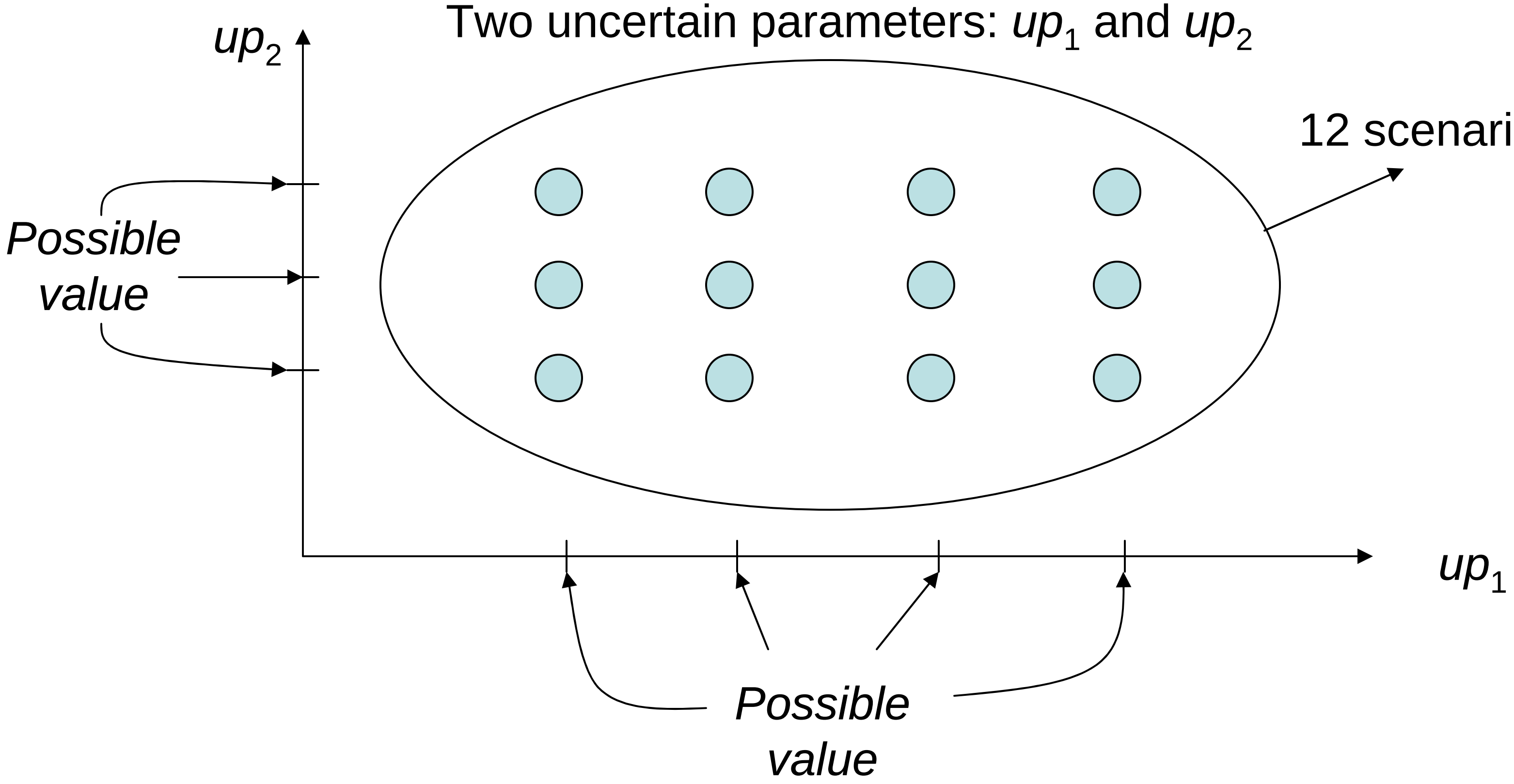
(MILP1 Model for Scenario ω)

*Not really an effective method when dealing with a large number of scenarios.

Research Question?

- How to effectively solve the problem with finitely large number of scenarios when scenarios are nicely designed?

The design of scenarios is a full factorial design.
(Each uncertain parameter independently takes its values from a finite set of discrete real values.)



How Large the Number of Scenarios can be with this Full Factorial Design?

*Assume that each uncertain parameter take its value from only 3 possible real values.

# Random Parameters	# Scenarios	# Random Parameters	# Scenarios
1	3	16	43,046,721
2	9	17	129,140,163
3	27	18	387,420,489
4	81	19	1,162,261,467
5	243	20	3,486,784,401
6	729	21	10,460,353,203
7	2,187	22	31,381,059,609
8	6,561	23	94,143,178,827
9	19,683	24	2.8243E+11
10	59,049	25	8.47289E+11
11	177,147	26	2.54187E+12
12	531,441	27	7.6256E+12
13	1,594,323	28	2.28768E+13
14	4,782,969	29	6.86304E+13
15	14,348,907	30	2.05891E+14

Our Ideas of the Effective Algorithm

- The algorithm should not required O^*_ω information for all scenarios.

- The algorithm should not blindly solve the full MILP2 model.

- The algorithm should be able to generate and identify a small subset of necessary scenarios required for solving the full problem.

- The algorithm should be able to generate a global optimal (or ε–optimal) robust solution of the full problem within a finite number of iterations.

Characteristic of Our Algorithm

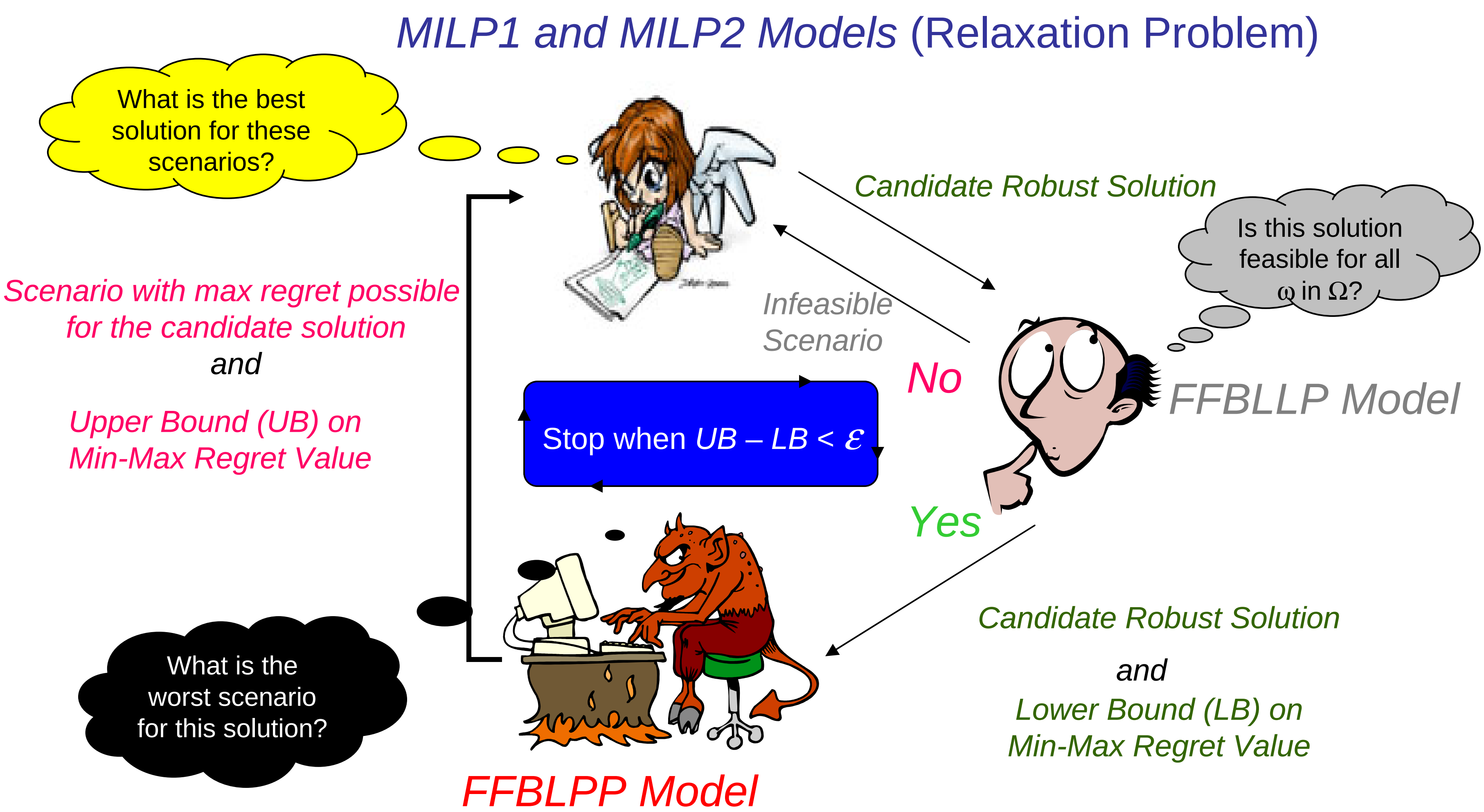
- The algorithm iteratively solves a small MILP2 model by considering only a small subset of all possible scenarios.

- The algorithm requires O^{*}_{ω} information only for a small subset of scenarios.

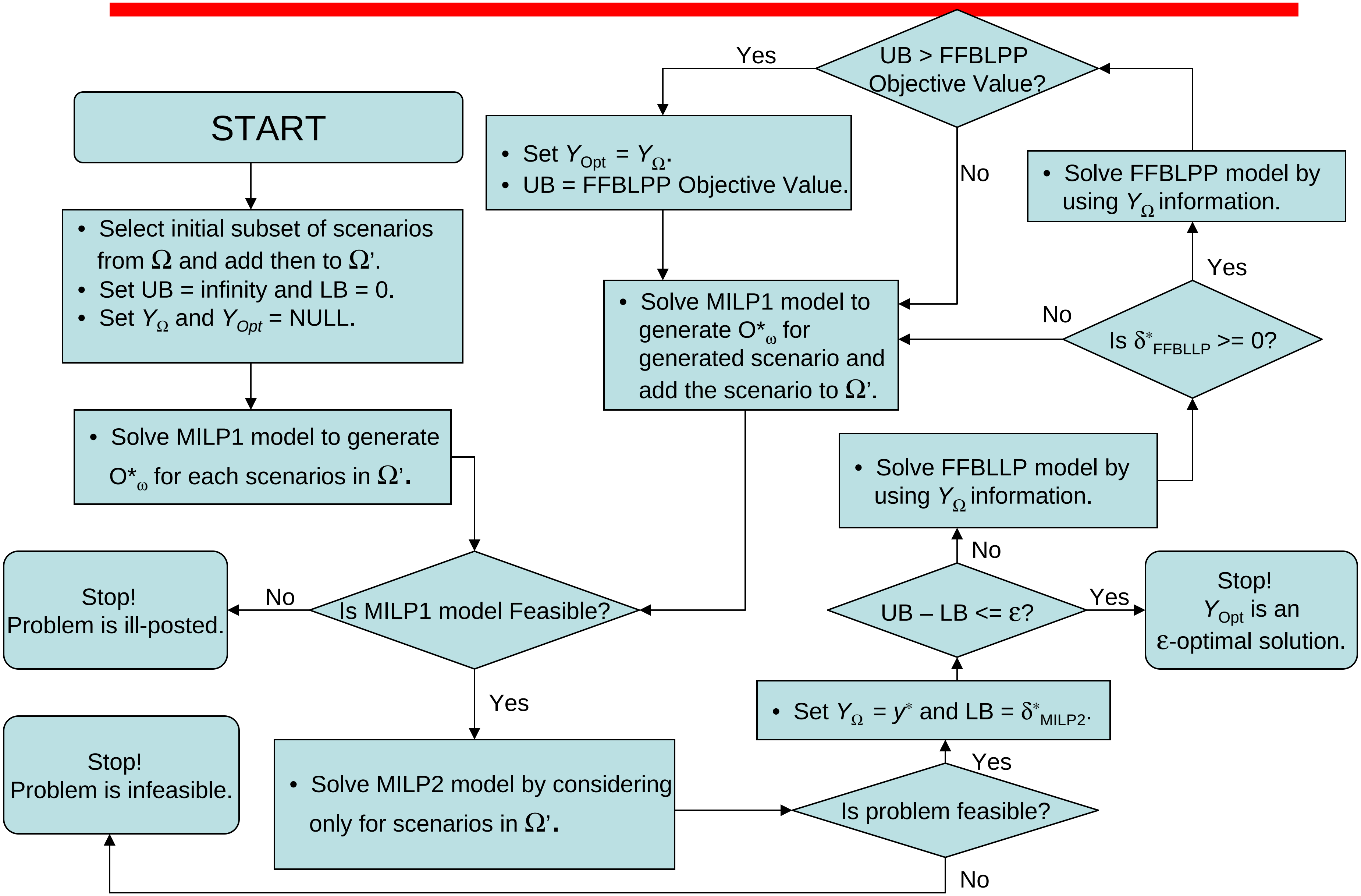
- The algorithm uses bi-level optimization models to generating necessary scenarios required for solving the full problem.

Result = Huge Saving in Computational Time

Full Factorial Robust Algorithm



Algorithm Flow Chart



FFBLLP Model

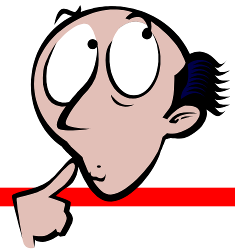
Purpose:

- To check if the candidate robust solution Y_Ω is feasible for all possible scenarios.
- If it is not, the model will identify the most infeasible scenario of the solution Y_Ω.

Initial Structure of the FFBLLP model:

$$
\begin{aligned}
&\underset{\vec{p}}{\text{minimize}} \quad \delta \\
&\text{s.t.} \quad \left.\begin{array}{l} p_k \in \{p_{k(0)}, p_{k(1)}, \ldots, p_{k(m_k-1)}\} \\ \text{where } p_{k(0)} \le p_{k(1)} \le \ldots \le p_{k(m_k-1)} \end{array}\right\} \quad \forall k \in \{1,2,\ldots,|\vec{p}|\} \\
&\quad \left.\begin{array}{l} P_{5ij} \in \{P_{5ij(0)}, P_{5ij(1)}, \ldots, P_{5ij(m_{ij}-1)}\} \\ \text{where } P_{5ij(0)} \le P_{5ij(1)} \le \ldots \le P_{5ij(m_{ij}-1)} \end{array}\right\} \quad \forall i \in \{1,2,\ldots,|\vec{s}|\} \quad \forall j \in \{1,2,\ldots,|\vec{x}|\} \\
&\quad \underset{\vec{x},\vec{s},\delta}{\text{maximize}} \quad \delta \\
&\quad \text{s.t.} \quad P_5\vec{x} \pm \vec{s} = \vec{p}_3 + P_2 Y_\Omega = B\vec{p} \\
&\quad\quad\quad \delta\vec{1} \le \vec{s} \\
&\quad\quad\quad \vec{x} \ge \vec{0}
\end{aligned}
$$

FFBLLP Model



Lemma 1:

The *FFBLLP* model has at least one optimal solution p^* and P^*_5 in which each element of p^* and P^*_5 takes on a value at one of its bounds.

$$\text{Minimize} \quad \delta$$

$$\text{s.t.} \quad \begin{pmatrix} p_k = p_{k(0)} + (p_{k(m_k-1)} - p_{k(0)}) b i_k \\ b i_k \in \{0,1\} \end{pmatrix} Ind_k \Bigg\} \qquad \forall k$$

$$\begin{pmatrix} P_{5ij} = P_{5ij(0)} + (P_{5ij(m_{5ij}-1)} - P_{5ij(0)}) b i^5_{ij} \\ b i^5_{ij} \in \{0,1\} \end{pmatrix} Ind^5_{ij} \Bigg\} \qquad \forall i, j$$

$$\sum_{\forall j} P^*_{5ij} x_j + \sum_{\forall j} Ind^5_{ij} P_{5ij} x_j (\pm) s_i = \sum_{\forall k} B_{ik} p^*_k + \sum_{\forall k} Ind_k B_{ik} p_k \qquad \forall i$$

$$\delta \le s_i \qquad \forall i$$

$$\sum_{\forall i} P^*_{5ij} w_{1i} + \sum_{\forall i} Ind^5_{ij} P_{5ij} w_{1i} \ge 0 \qquad \forall j$$

$$\sum_{\forall i} w_{2i} = 1$$

$$(\pm) w_{1i} - w_{2i} = 0 \qquad \forall i$$

$$\delta = \sum_{\forall i} \sum_{\forall k} B_{ik} p^*_k w_{1i} + \sum_{\forall i} \sum_{\forall k} Ind_k B_{ik} p_k w_{1i}$$

$$w_{2i} \ge 0 \quad \forall i, \quad x_j \ge 0 \quad \forall j$$

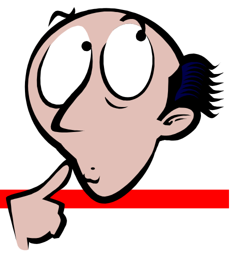
$$p^*_k = \begin{cases} \text{Preprocessed value of } p_k \text{ if } p_k \text{ can be preprocessed} \\ 0 \quad \text{Otherwise} \end{cases}$$

$$Ind_k = \begin{cases} 1 \quad \text{if } p_k \text{ value cannot be predetermined.} \\ 0 \quad \text{otherwise} \end{cases}$$

$$P^*_{5ij} = \begin{cases} \text{Preprocessed value of } P_{5ij} \text{ if } P_{5ij} \text{ can be preprocessed} \\ 0 \quad \text{Otherwise} \end{cases}$$

$$Ind^5_{ij} = \begin{cases} 1 \quad \text{if } P_{5ij} \text{ value cannot be predetermined.} \\ 0 \quad \text{otherwise} \end{cases}$$

FFBLLP Model



<u>*Final Transformation Steps*</u>:

The pervious version of the FFBLLP model still requires the following final transformation steps by using the results from *Lemma* 1.

$$P_{5ij}x_j \equiv PX_{5ij} \text{ where } P_{5ij(0)}x_j \le PX_{5ij} \le P_{5ij(m_{5ij}-1)}x_j \text{ and } P_{5ij(m_{5ij}-1)}x_j - M(1-bi_{ij}^5) \le PX_{5ij} \le P_{5ij(0)}x_j + Mbi_{ij}^5$$

$$P_{5ij}w_{1i} \equiv (\pm)PW_{5ij} \text{ where } P_{5ij(0)}w_{2i} \le PW_{5ij} \le P_{5ij(m_{5ij}-1)}w_{2i} \text{ and } P_{5ij(m_{5ij}-1)}w_{2i} - M(1-bi_{ij}^5) \le PW_{5ij} \le P_{5ij(0)}w_{2i} + Mbi_{ij}^5$$

$$p_k w_{1i} \equiv (\pm)PW_{ki} \text{ where } p_{k(0)}w_{2i} \le PW_{ki} \le p_{k(m_k-1)}w_{2i} \text{ and } p_{k(m_k-1)}w_{2i} - M(1-bi_k) \le PW_{ki} \le p_{k(0)}w_{2i} + Mbi_k$$

FFBLPP Model

Purpose:

- To find the scenario with the maximum regret value for the candidate robust solution Y_Ω.

Initial Structure of the FFBLPP model:

$$\max_{x_1, y_1, p} \{ \vec{p}_4^{\,T} \vec{x}_1 + \vec{p}_1^{\,T} \vec{y}_1 - \max_{x_2} (\vec{p}_4^{\,T} \vec{x}_2 + \vec{p}_1^{\,T} Y_\Omega) \}$$

$$s.t. \quad P_5 \vec{x}_1 - P_2 \vec{y}_1 \le \vec{p}_3$$

$$P_5 \vec{x}_2 - P_2 Y_\Omega \le \vec{p}_3$$

$$\left. \begin{array}{l} p_{lk} \in \{ p_{lk(0)}, p_{lk(1)}, \ldots, p_{lk(m_{lk}-1)} \} \\ \text{where } p_{lk(0)} \le p_{lk(1)} \le \ldots \le p_{lk(m_{lk}-1)} \end{array} \right\} \quad \forall l = 1, 3, 4 \quad \forall k \in \{1, 2, \ldots, |\vec{p}_l|\}$$

$$\left. \begin{array}{l} P_{2ij} \in \{ P_{2ij(0)}, P_{2ij(1)}, \ldots, P_{2ij(m_{2ij}-1)} \} \\ \text{where } P_{2ij(0)} \le P_{2ij(1)} \le \ldots \le P_{2ij(m_{2ij}-1)} \end{array} \right\} \quad \forall i \in \{1, 2, \ldots, |\vec{p}_3|\} \quad \forall j \in \{1, 2, \ldots, |\vec{y}_1|\}$$

$$\left. \begin{array}{l} P_{5ij} \in \{ P_{5ij(0)}, P_{5ij(1)}, \ldots, P_{5ij(m_{5ij}-1)} \} \\ \text{where } P_{5ij(0)} \le P_{5ij(1)} \le \ldots \le P_{5ij(m_{5ij}-1)} \end{array} \right\} \quad \forall i \in \{1, 2, \ldots, |\vec{p}_3|\} \quad \forall j \in \{1, 2, \ldots, |\vec{x}|\}$$

$$\forall \vec{x}_1, \vec{x}_2 \ge \vec{0} \quad \vec{y}_1 \in \{0,1\}^{|\vec{y}_1|}$$

FFBLPP Model

$$\max_{\bar{x}_1,\bar{y}_1,\bar{p}}\{\sum_{\forall j} p_{4j}x_{1j} + \sum_{\forall k} p_{1k}y_{1k} - \sum_{\forall j} p_{4j}x_{2j} - \sum_{\forall k} p_{1k}Y_{\Omega k}\}$$

$$s.t.\quad \sum_{\forall j} P_{5ij}x_{1j} - \sum_{\forall k} P_{2ik}y_{1k} \le p_{3i} \quad \forall i$$

$$\sum_{\forall j} P_{5ij}x_{2j} - \sum_{\forall k} P_{2ik}Y_{\Omega k} \le p_{3i} \quad \forall i$$

$$\sum_{\forall i} P_{5ij}w_i \ge p_{4j} \quad \forall j$$

$$\sum_{\forall i}\left(p_{3i} + \sum_{\forall k} P_{2ik}Y_{\Omega k}\right)w_i = \sum_{\forall j} p_{4j}x_{2j}$$

Dual Constraints of the follower problem and the strong duality constraint.

$$p_{1k} \in \{p_{1k(0)}, p_{1k(1)}, \dots, p_{1k(m_{1k}-1)}\} \quad \forall k \qquad P_{2ik} \in \{P_{2ik(0)}, P_{2ik(1)}, \dots, P_{2ik(m_{2ik}-1)}\} \quad \forall i \forall k$$

$$p_{3i} \in \{p_{3i(0)}, p_{3i(1)}, \dots, p_{3i(m_{3i}-1)}\} \quad \forall i \qquad p_{4j} \in \{p_{4j(0)}, p_{4j(1)}, \dots, p_{4j(m_{4j}-1)}\} \quad \forall j$$

$$P_{5ij} \in \{P_{5ij(0)}, P_{5ij(1)}, \dots, P_{5ij(m_{5ij}-1)}\} \quad \forall i \forall j$$

$$x_{1j}, x_{2j}, w_i \ge 0 \qquad y_{1k} \in \{0,1\} \qquad \forall i, j, k$$

where $p_{1k(0)} \le p_{1k(1)} \le \dots \le p_{1k(m_{1k}-1)}$, $P_{2ik(0)} \le P_{2ik(1)} \le \dots \le P_{2ik(m_{2ik}-1)}$,

$p_{3i(0)} \le p_{3i(1)} \le \dots \le p_{3i(m_{3i}-1)}$, $p_{4j(0)}, p_{4j(1)}, \dots, p_{4j(m_{4j}-1)}$, and $P_{5ij(0)} \le P_{5ij(1)} \le \dots \le P_{5ij(m_{5ij}-1)}$

FFBLPP Model

$$\max_{\bar{x}_1,\bar{y}_1,\bar{p}}\Big\{\sum_{\forall j} Ind_{4j}PX_{41j}+\sum_{\forall j} p^*_{4j}x_{1j}+\sum_{\forall k} p^*_{1k}y_{1k}-\sum_{\forall j} Ind_{4j}PX_{42j}-\sum_{\forall j} p^*_{4j}x_{2j}-\sum_{\forall k} p^*_{1k}Y_{\Omega k}\Big\}$$

$$s.t.\quad \sum_{\forall j} Ind_{5ij}PX_{51ij}+\sum_{\forall j} P^*_{5ij}x_{1j}-\sum_{\forall k} Ind_{2ik}PY_{21ik}-\sum_{\forall k} P^*_{2ik}y_{1k}\le\left(Ind_{3i}p_{3i}+p^*_{3i}\right)\quad\forall i$$

$$\sum_{\forall j} Ind_{5ij}PX_{52ij}+\sum_{\forall j} P^*_{5ij}x_{2j}-\sum_{\forall k} Ind_{2ik}P_{2ik}Y_{\Omega k}-\sum_{\forall k} P^*_{2ik}Y_{\Omega k}\le\left(Ind_{3i}p_{3i}+p^*_{3i}\right)\quad\forall i$$

$$\sum_{\forall i} Ind_{5ij}PW_{5ij}+\sum_{\forall i} P^*_{5ij}w_i\ge\left(Ind_{4j}p_{4j}+p^*_{4j}\right)\quad\forall j$$

$$\sum_{\forall i} Ind_{3i}PW_{3i}+\sum_{\forall i} p^*_{3i}w_i+\sum_{\forall k} Ind_{2ik}PW_{2ik}Y_{\Omega k}+\sum_{\forall k} P^*_{2ik}w_iY_{\Omega k}=\sum_{\forall j} Ind_{4j}PX_{42j}+\sum_{\forall j} p^*_{4j}x_{2j}$$

$$\left.\left(\begin{array}{l}
PY_{21ik}-P_{2ik}+P_{2ik(0)}(1-y_{1k})\le 0,\quad -PY_{21ik}+P_{2ik}-P_{2ik(m_{2ik}-1)}(1-y_{1k})\le 0\\
P_{2ik(0)}y_{1k}\le PY_{21ik}\le P_{2ik(m_{2ik}-1)}y_{1k},\quad P_{2ik}\le P_{2ik(0)}+y_{1k}(P_{2ik(m_{2ik}-1)}-P_{2ik(0)})\\
P_{2ik}=P_{2ik(0)}+\sum_{s=1}^{m_{2ik}-1}Co_{2ik(s)}Z_{2ik(s)},\quad \sum_{l\in tabu} bi^2_{ik(l)}\le|tabu|-1\quad\forall tabu\in TABU_{m_{2ik}}\\
\left.\begin{array}{l}
Z_{2ik(s)}=bi^2_{ik(d+1)}\in\{0,1\}\ \text{ if } s=2^d\ \exists d\in\mathbb{Z}^+\cup\{0\}\\
\left.\begin{array}{l}
Z_{2ik(s)}\le bi^2_{ik(d)},\quad Z_{2ik(s)}\le Z_{2ik(s-2^{d-1})},0\le Z_{2ik(s)}\le 1\\
Z_{2ik(s)}\ge\sum_{\forall l\in S_{(s)}} bi^2_{ik(l)}-|S_{(s)}|+1
\end{array}\right\}\ \text{if } 2^{d-1}<s<2^d\ \exists d\in\mathbb{Z}^+\cup\{0\}
\end{array}\right\}\ \forall s=0,1,\ldots,m_{2ik}-1\\
PW_{2ik}=P_{2ik(0)}w_i+\sum_{s=1}^{m_{2ik}-1}Co_{2ik(s)}ZW_{2ik(s)}\\
0\le ZW_{2ik(s)}\le w_i^UZ_{2ik(s)},\quad ZW_{2ik(s)}\le w_i-w_i^L(1-Z_{2ik(s)}),\quad ZW_{2ik(s)}\ge w_i-w_i^U(1-Z_{2ik(s)})\quad\forall s=0,1,\ldots,m_{2ik}-1
\end{array}\right)Ind_{2ik}\right\}\quad\forall i,k$$

$$\left.\left(\begin{array}{l}
p_{3i}=p_{3i(0)}+\sum_{s=1}^{m_{3i}-1}Co_{3i(s)}Z_{3i(s)},\quad\sum_{l\in tabu} bi^3_{i(l)}\le|tabu|-1\quad\forall tabu\in TABU_{m_{3i}}\\
\left.\begin{array}{l}
Z_{3i(s)}=bi^3_{i(d+1)}\in\{0,1\}\ \text{ if } s=2^d\ \exists d\in\mathbb{Z}^+\cup\{0\}\\
\left.\begin{array}{l}
Z_{3i(s)}\le bi^3_{i(d)},\quad Z_{3i(s)}\le Z_{3i(s-2^{d-1})},0\le Z_{3i(s)}\le 1\\
Z_{3i(s)}\ge\sum_{\forall l\in S_{(s)}} bi^3_{i(l)}-|S_{(s)}|+1
\end{array}\right\}\ \text{if } 2^{d-1}<s<2^d\ \exists d\in\mathbb{Z}^+\cup\{0\}
\end{array}\right\}\ \forall s=0,1,\ldots,m_{3i}-1\\
PW_{3i}=p_{3i(0)}w_i+\sum_{s=1}^{m_{3i}-1}Co_{3i(s)}ZW_{3i(s)}\\
0\le ZW_{3i(s)}\le w_i^UZ_{3i(s)},\quad ZW_{3i(s)}\le w_i-w_i^L(1-Z_{3i(s)}),\quad ZW_{3i(s)}\ge w_i-w_i^U(1-Z_{3i(s)})\quad\forall s=0,1,\ldots,m_{3i}-1
\end{array}\right)Ind_{3i}\right\}\quad\forall i$$

FFBLPP Model

$$
\left.\begin{pmatrix}
PX_{41j} - p_{4j(m_{4j}-1)} x_{1j} \le 0, \quad -PX_{41j} + p_{4j(0)} x_{1j} \le 0 \\
-PX_{41j} + p_{4j(m_{4j}-1)} x_{1j} - (p_{4j(m_{4j}-1)} x_{1j}^{U} + |\min(0, p_{4j(0)})|\, x_{1j}^{U})(1 - bi_j^4) \le 0 \\
PX_{41j} - p_{4j(0)} x_{1j} - (p_{4j(m_{4j}-1)} x_{1j}^{U} + |\min(0, p_{4j(0)})|\, x_{1j}^{U}) bi_j^4 \le 0 \\
PX_{42j} - p_{4j(m_{4j}-1)} x_{2j} \le 0, \quad -PX_{42j} + p_{4j(0)} x_{2j} \le 0 \\
-PX_{42j} + p_{4j(m_{4j}-1)} x_{2j} - (p_{4j(m_{4j}-1)} x_{2j}^{U} + |\min(0, p_{4j(0)})|\, x_{2j}^{U})(1 - bi_j^4) \le 0 \\
PX_{42j} - p_{4j(0)} x_{2j} - (p_{4j(m_{4j}-1)} x_{2j}^{U} + |\min(0, p_{4j(0)})|\, x_{2j}^{U})\, bi_j^4 \le 0 \\
p_{4j} = p_{4j(0)} + (p_{4j(m_{4j}-1)} - p_{4j(0)}) bi_j^4, \quad bi_j^4 \in \{0,1\}
\end{pmatrix} Ind_{4j} \right\} \quad \forall j
$$

$$
\left.\begin{pmatrix}
PX_{51ij} = P_{5ij(0)} x_{1j} + \sum_{s=1}^{m_{5ij}-1} Co_{5ij(s)} ZX_{51ij(s)}, \; PX_{52ij} = P_{5ij(0)} x_{2j} + \sum_{s=1}^{m_{5ij}-1} Co_{5ij(s)} ZX_{52ij(s)}, \; \sum_{l \in tabu} bi_{ij(l)}^5 \le |tabu| - 1 \quad \forall tabu \in TABU_{m_{5ij}} \\
0 \le ZX_{51ij(s)} \le x_{1j}^U Z_{5ij(s)}, \; ZX_{51ij(s)} \le x_{1j} - x_{1j}^L (1 - Z_{5ij(s)}), \; ZX_{51ij(s)} \ge x_{1j} - x_{1j}^U (1 - Z_{5ij(s)}) \quad \forall s = 0,1,\dots, m_{5ij} - 1 \\
0 \le ZX_{52ij(s)} \le x_{2j}^U Z_{5ij(s)}, \; ZX_{52ij(s)} \le x_{2j} - x_{2j}^L (1 - Z_{5ij(s)}), \; ZX_{52ij(s)} \ge x_{2j} - x_{2j}^U (1 - Z_{5ij(s)}) \quad \forall s = 0,1,\dots, m_{5ij} - 1 \\
\left.\begin{array}{l}
Z_{5ij(s)} = bi_{ij(d+1)}^5 \in \{0,1\} \;\; \text{if } s = 2^d \; \exists d \in \mathbb{Z}^+ \cup \{0\} \\
\left.\begin{array}{l}
Z_{5ij(s)} \le bi_{ij(d)}^5, \; Z_{5ij(s)} \le Z_{5ij(s-2^{d-1})}, 0 \le Z_{5ij(s)} \le 1 \\
Z_{5ij(s)} \ge \sum_{\forall l \in S_{(s)}} bi_{ij(l)}^5 - |S_{(s)}| + 1
\end{array}\right\} \text{if } 2^{d-1} < s < 2^d \;\; \exists d \in \mathbb{Z}^+ \cup \{0\}
\end{array}\right\} \quad \forall s = 0,1,\dots, m_{5ij} - 1 \\
PW_{5ij} = P_{5ij(0)} w_i + \sum_{s=1}^{m_{5ij}-1} Co_{5ij(s)} ZW_{5ij(s)} \\
0 \le ZW_{5ij(s)} \le w_i^U Z_{5ij(s)}, \; ZW_{5ij(s)} \le w_i - w_i^L (1 - Z_{5ij(s)}), \; ZW_{5ij(s)} \ge w_i - w_i^U (1 - Z_{5ij(s)}) \quad \forall s = 0,1,\dots, m_{5ij} - 1
\end{pmatrix} Ind_{5ij} \right\} \quad \forall i, j
$$

$$
x_{1j}, x_{2j}, w_i \ge 0 \qquad y_{1k} \in \{0,1\} \qquad \forall i, j, k
$$

FFBLPP Model

$$Ind_{2ik} = \begin{cases} 1 & \text{if } P_{2ik} \text{ value cannot be predetermined.} \\ 0 & \text{otherwise} \end{cases}$$

$$Ind_{3i} = \begin{cases} 1 & \text{if } p_{3i} \text{ value cannot be predetermined.} \\ 0 & \text{otherwise} \end{cases}$$

$$Ind_{4j} = \begin{cases} 1 & \text{if } p_{4j} \text{ value cannot be predetermined.} \\ 0 & \text{otherwise} \end{cases}$$

$$Ind_{5ij} = \begin{cases} 1 & \text{if } P_{5ij} \text{ value cannot be predetermined.} \\ 0 & \text{otherwise} \end{cases}$$

$$P^*_{2ik} = \begin{cases} \text{Preprocessed value of } P_{2ik} & \text{if } P_{2ik} \text{ can be preprocessed} \\ 0 & \text{Otherwise} \end{cases}$$

$$p^*_{3i} = \begin{cases} \text{Preprocessed value of } p_{3i} & \text{if } p_{3i} \text{ can be preprocessed} \\ 0 & \text{Otherwise} \end{cases}$$

$$p^*_{4j} = \begin{cases} \text{Preprocessed value of } p_{4j} & \text{if } p_{4j} \text{ can be preprocessed} \\ 0 & \text{Otherwise} \end{cases}$$

$$P^*_{5ij} = \begin{cases} \text{Preprocessed value of } P_{5ij} & \text{if } P_{5ij} \text{ can be preprocessed} \\ 0 & \text{Otherwise} \end{cases}$$

Full Factorial Robust Algorithm

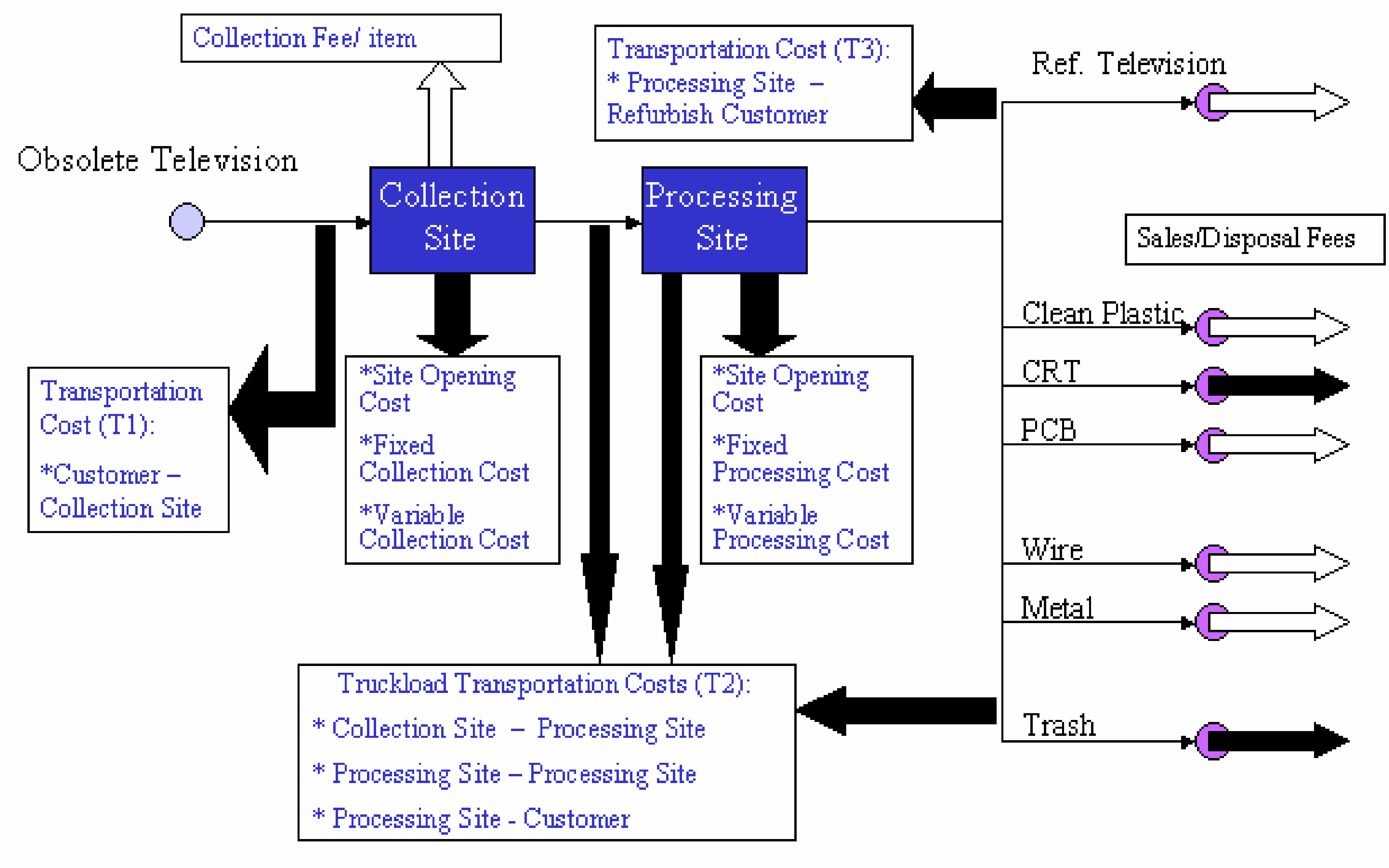
Lemma 2: The algorithm presented terminates in finite number of steps.
After the algorithm terminated with $\varepsilon = 0$, it has either detected infeasibility or has found an optimal robust solution to the original problem

Now let see how this proposed algorithm work in the real problem.

Robust RPS Infrastructure for Television Recycle in GA

- 12 Municipal collection sites
- 9 Commercial processing sites (A)

Cash Flow Diagram of the Model



Problem Size without Uncertainty

Model Type	Number of Constraints	Number of Continuous Variables	Number of Binary Variables
MILP1	14,182	11,843	1,180

Problem Size with Uncertainty

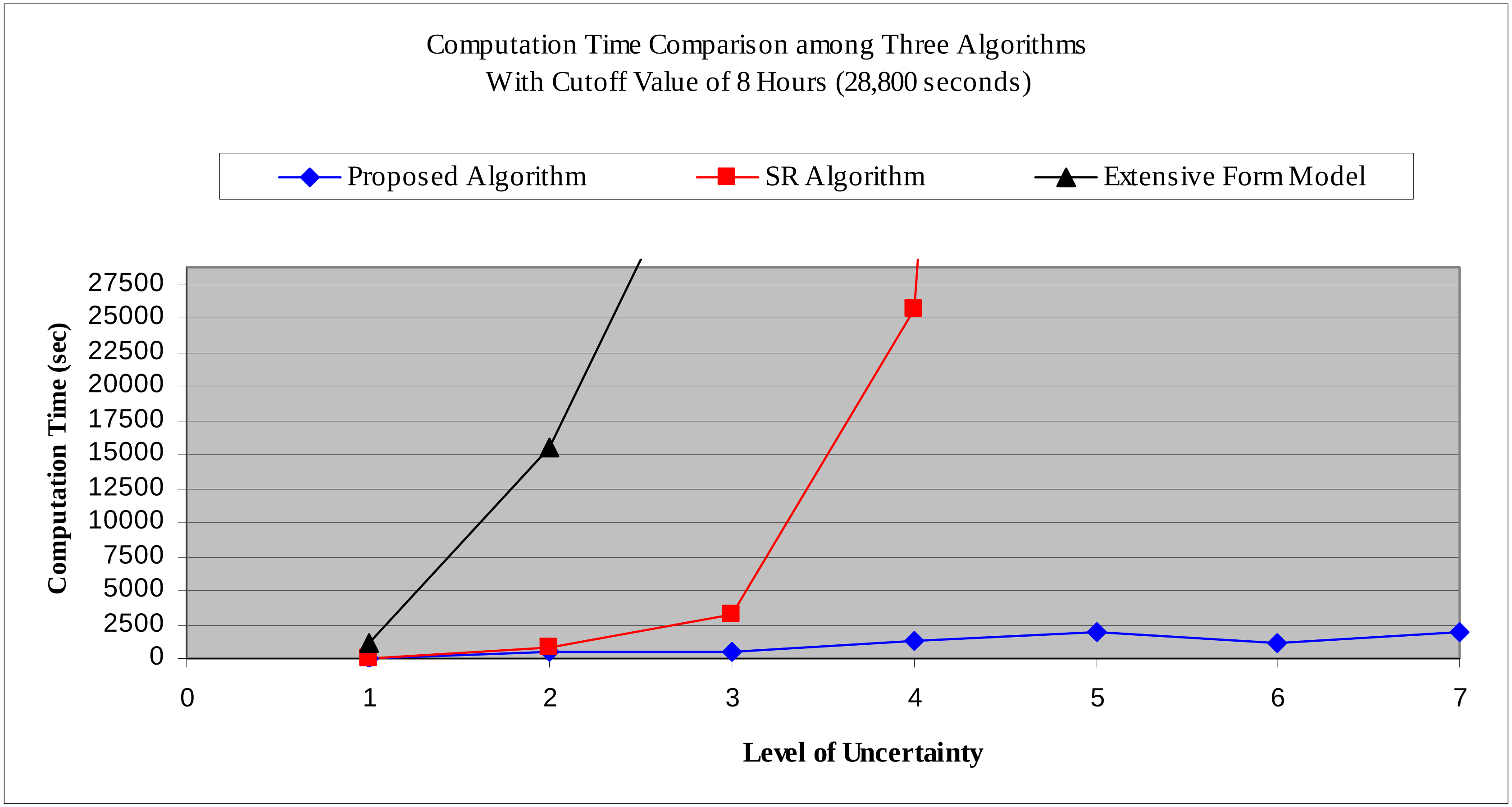
Uncertainty Level	Number of Possible Scenarios	Number of *MILP*2 Constraints	Number of *MILP*2 Continuous Variables	Number of *MILP*2 Binary Variables
1	8	102,396	94,744	1,180
2	64	808,108	757,952	1,180
3	512	6,453,804	6,063,616	1,180
4	4,096	51,619,372	48,508,928	1,180
5	32,768	412,943,916	388,071,424	1,180
6	262,144	3,303,540,268	3,104,571,392	1,180
7	2,097,152	26,428,311,084	24,836,571,136	1,180

For uncertainty level 3-7, the direct method failed to solve the problem using C++ with CPLEX 9.0 on Pentium (R) 4 CPU 3.6 GHz with 2 GB RAM . (Still running after 8 hours)

Performance of the Proposed Algorithm

Uncertainty Level	Total # Scenarios	# Scenarios Generated	Ratio Between # Scenarios Generated and Total # Scenarios	Min-Max Regret	Time (sec) Proposed Algorithm	Time (sec) Direct Method
1	8	2	25%	5,244.75	58.50	1,186.88
2	64	4	6.25%	42,397.84	506.30	15,504.51
3	512	4	0.78%	42,397.84	516.29	N/A
4	4,096	6	0.15%	46,756.29	1,246.36	N/A
5	32,768	7	0.02%	51,918.70	1,909.99	N/A
6	262,144	5	0.0019%	52,100.33	1,084.43	N/A
7	2,097,152	6	0.0003%	53,864.33	1,947.89	N/A

Performance of the Proposed Algorithm



Research Contribution

- ***Theoretical Contribution***

  Innovative approaches to generate Min-Max regret robust solution when each uncertain parameter independently takes its value from a finite set of real values and the scenarios design is a full factorial design.

- ***Practical Contribution***

  Practical approaches for large-scale robust planning problem under uncertainty.

Special Thanks

Dr. Jane C. Ammons
Industrial & Systems Engineering

Dr. Matthew J. Realff
Chemical & Biomolecular Engineering

Questions

Thank you so much for coming to my presentation. Please have a wonderful day.

Please feel free to ask questions. I will do my best to answer all of them.