

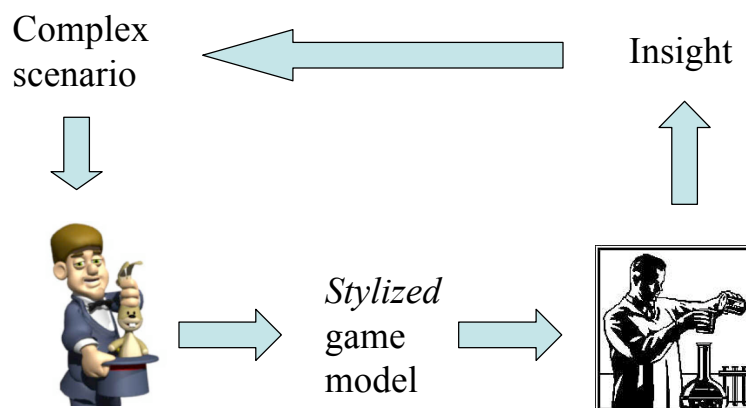


## Toward Practical Strategic (Game-Theoretic) Reasoning

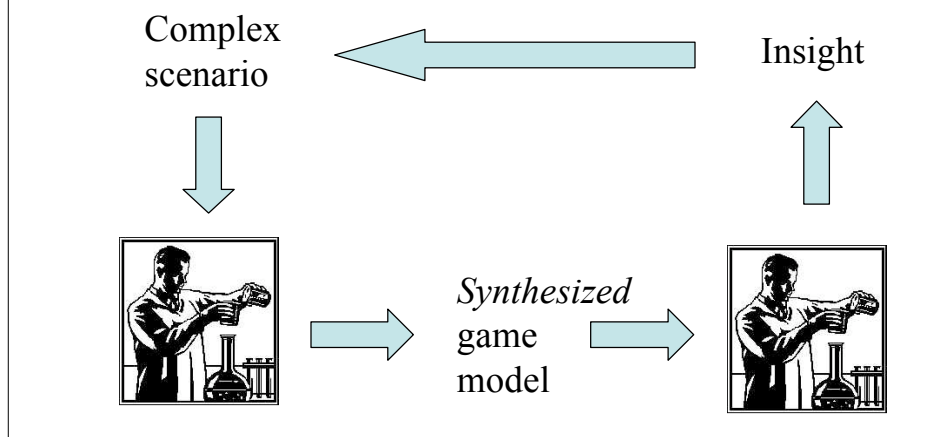
Michael Wellman

based on joint work with JK MacKie-Mason,  
S Singh, C Kiekintveld, A Osepashvili,  
DM Reeves, Y Vorobeychik, ...

## Game Theory in Strategic Reasoning



## Widening the Scope of Game-Theoretic Analysis



## An Empirical Game-Theoretic Methodology

Three steps (iterate)

1. Parametrize strategy space
2. Estimate “empirical game”
3. Solve empirical game; analyze results

- Shares common elements with much prior work
- Recently applied to TAC games, SAA, Chaturanga,...

# 1. Parametrize Strategy Space

- Transform direct policy representation (e.g., game subtree) to implicit form based on local decision method
- Examples:
  - SAA: “Straightforward bidding” as baseline, with parameters controlling sunk-cost awareness, price prediction methods
  - TAC [travel-shopping](#): Skeletal decision cycle, modified by controls for timing of flight purchases, shading of hotel bids, entertainment trading policy
  - TAC [supply chain](#): Abstract strategic procurement into qualitative behavior categories
  - [Chaturanga](#): *next...*

## Chaturanga Strategies

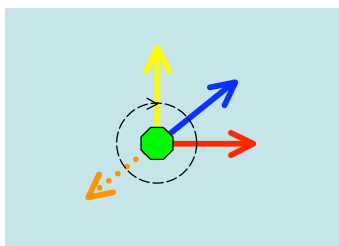
- 4-player minimax to fixed depth ( $\epsilon$ -greedy)
- Static evaluator:
  - $F(\text{features, weights}) = \text{value}$
  - Defined ~60 features/weights so far, assessing relative strengths of each players' position



- Using single-agent learning methods to identify candidate strategies
  - play against fixed opponents
  - take snapshot when performance plateaus
  - explore different context based on candidate set
  - Idea: factor out non-strategic strategy search

## “Attack” Angle

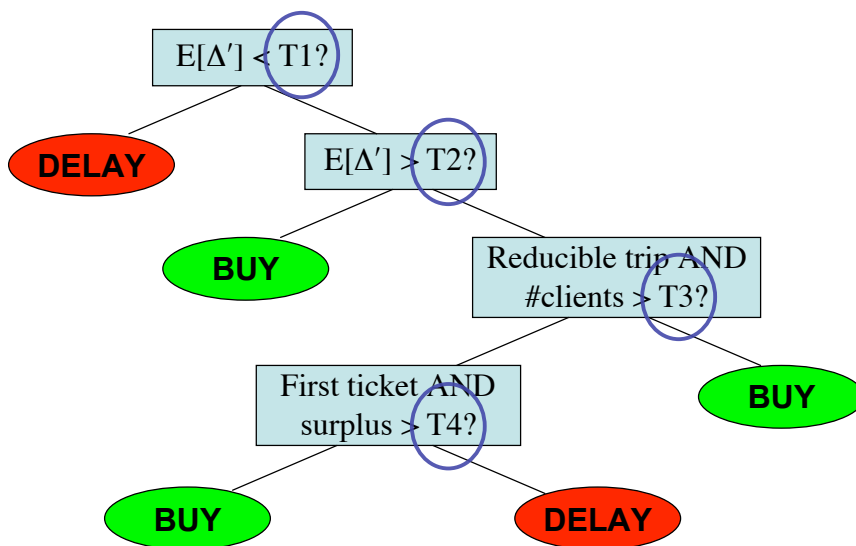
- Example strategic parameter
- Controls relative weighting of position comparison to respective opponents



Preliminary findings:

- Player pointed to by many opponents at great disadvantage
- Player clockwise from target key beneficiary

## Flight Purchase Decision Tree





## Control Variates

- Standard variance reduction technique
- Example: Demand Adjusted Profit (DAP) in TAC/SCM
  - Small numbers of games (30 / profile)
  - Reduce variance by accounting for influence of customer demand
    - Avg Q as *control variate*
    - DAP Estimator

$$E[x] = \int_{\bar{Q}} E[x|\bar{Q}]Pr(\bar{Q})d\bar{Q}$$

## Hierarchical Game Reduction

- $p$ -player reduced version of symmetric game  $\Gamma$

$$\Gamma \downarrow_p = \langle p, S, \hat{u}(\cdot) \rangle$$

where

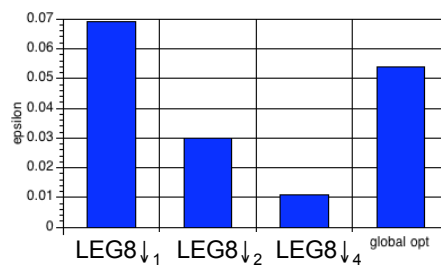
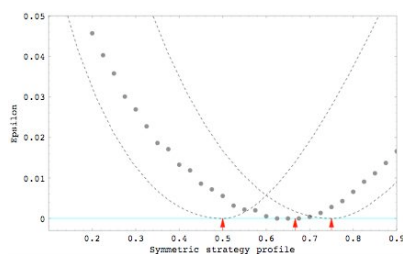
$$\hat{u}_i(s_1, \dots, s_p) = u_{q,i}(\underbrace{s_1, \dots, s_2, \dots, s_p, \dots}_{q}, \dots)$$

**Premise:** Reduced game often a good approximation of original, with dramatically smaller profile space.

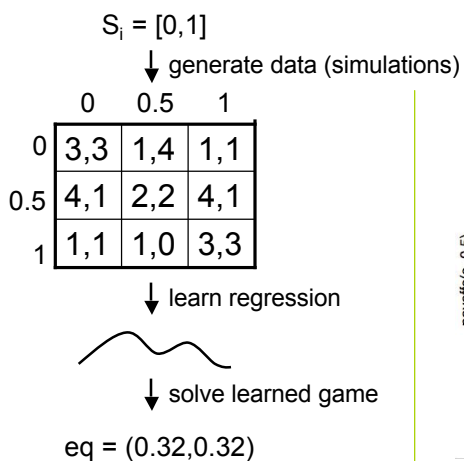
Empirical and theoretical evidence to date from several games: FPSB, BO, LEG, TAC travel

## Why Trust Reduced-Game Results?

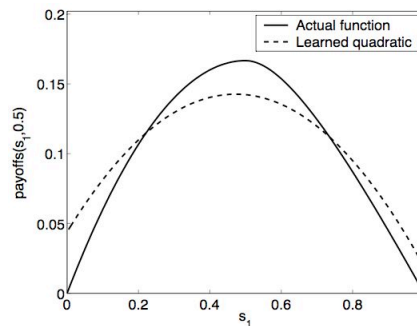
- Claim: Equilibria in reduced game likely to be relatively stable in full game
- Evidence:
  - Random instances of **local-effect games** (LEGs)
  - **FPSB** auctions



## Payoff Function Regression



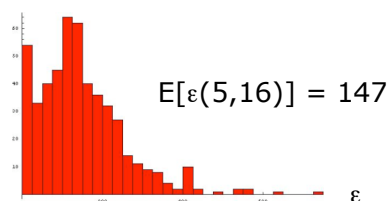
FPSB Example



### 3. Solve/Analyze Game

- Apply off-the-shelf game solvers
- Exploit structure (e.g., symmetry)
  - Replicator dynamics
  - Small-support search (Porter et al.)
- Sensitivity analysis

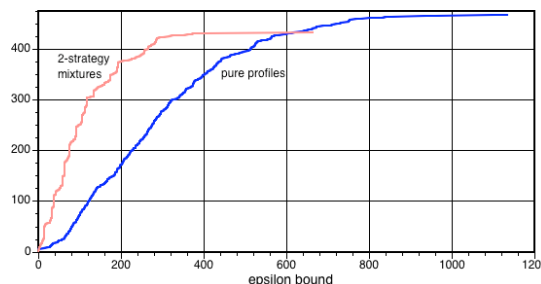
- Empirical game payoffs estimated from samples
- Assume independent,  $\sim N(\underline{x}, \underline{V})$



### Analyzing Incomplete Games

- Classify profile  $(s, s')$  wrt empirical game:
  - **Unevaluated:**  $\hat{u}$  is undefined
  - **Refuted:** o.w., and for some  $t$ ,  $\hat{u}(t, s') > \hat{u}(s, s')$  OR  $\hat{u}(t, s) > \hat{u}(s', s)$
  - **Candidate:** o.w., and for some  $t$ ,  $(t, s')$  OR  $(s, t)$  unevaluated
  - **Confirmed:** o.w.

- $TAC_{\downarrow 2}$  profiles
  - 467 evaluated
  - 5 confirmed PSNE
  - Most refuted  $\epsilon > 265$





## Recap

- Empirical game-theoretic analysis
- Employ simulation, estimation, regression, approximation, (many specific techniques and tricks)... to derive **empirical game**
- ... toward **practical** game-theoretic analysis for **routine** strategic reasoning

## References

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- Self-confirming price prediction for bidding in simultaneous ascending auctions, *UAI*, 2005.  
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- Empirical mechanism design: Methods, with application to a supply-chain scenario, *in prep.*