

Lecture slides for  
*Automated Planning: Theory and Practice*

# **Chapter 4**

## **State-Space Planning**

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# Motivation

- Nearly all planning procedures are search procedures
- Different planning procedures have different search spaces
  - ◆ Two examples:
- *State-space planning*
  - ◆ Each node represents a state of the world
    - » A plan is a path through the space
- *Plan-space planning*
  - ◆ Each node is a set of partially-instantiated operators, plus some constraints
    - » Impose more and more constraints, until we get a plan

# Outline

- State-space planning
  - ◆ Forward search
  - ◆ Backward search
  - ◆ Lifting
  - ◆ STRIPS
  - ◆ Block-stacking

# Forward-search( $O, s_0, g$ )

$s \leftarrow s_0$

$\pi \leftarrow$  the empty plan

loop

if  $s$  satisfies  $g$  then return  $\pi$

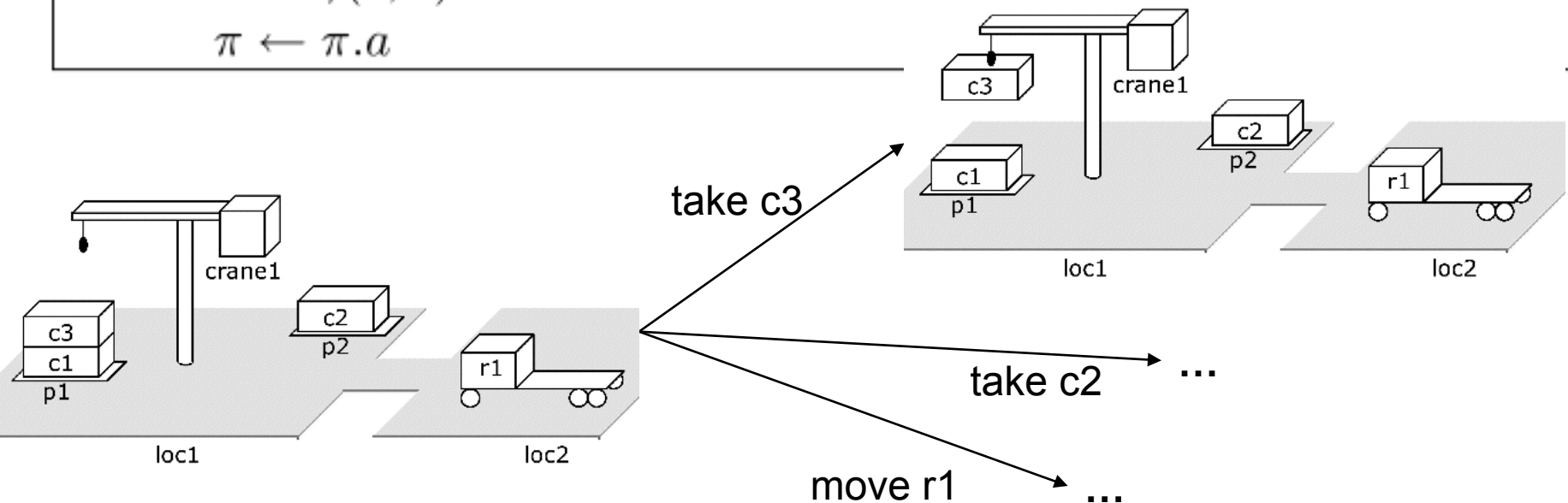
$E \leftarrow \{a \mid a \text{ is a ground instance an operator in } O,$   
and  $\text{precond}(a) \text{ is true in } s\}$

if  $E = \emptyset$  then return failure

nondeterministically choose an action  $a \in E$

$s \leftarrow \gamma(s, a)$

$\pi \leftarrow \pi.a$



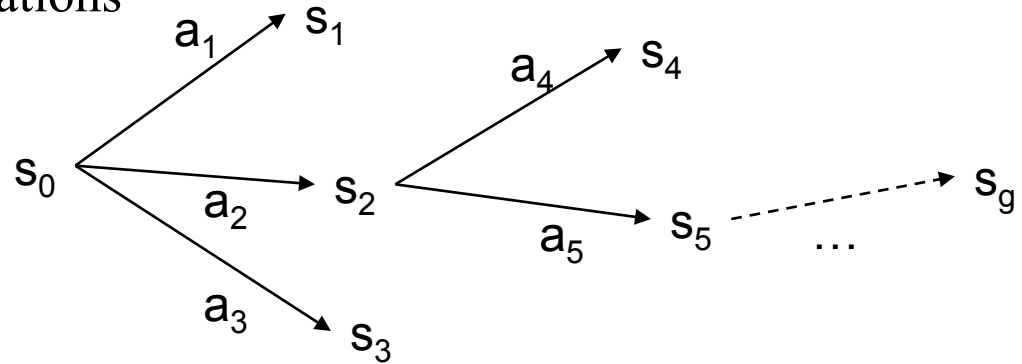
# Properties

- Forward-search is *sound*
  - ◆ for any plan returned by any of its nondeterministic traces, this plan is guaranteed to be a solution
- Forward-search also is *complete*
  - ◆ if a solution exists then at least one of Forward-search's nondeterministic traces will return a solution.

# Deterministic Implementations

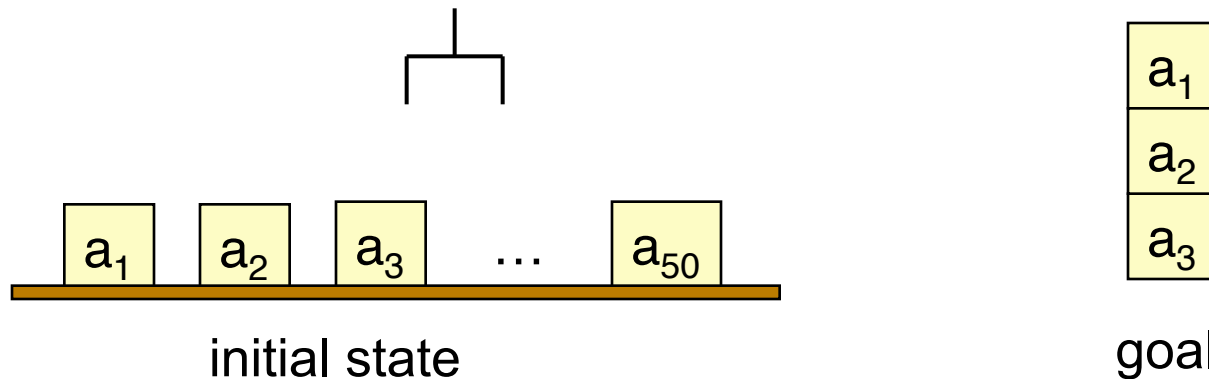
- Some deterministic implementations of forward search:

- ◆ breadth-first search
- ◆ depth-first search
- ◆ best-first search (e.g.,  $A^*$ )
- ◆ greedy search



- Breadth-first and best-first search are sound and complete
  - ◆ But they usually aren't practical because they require too much memory
  - ◆ Memory requirement is exponential in the length of the solution
- In practice, more likely to use depth-first search or greedy search
  - ◆ Worst-case memory requirement is linear in the length of the solution
  - ◆ In general, sound but not complete
    - » But classical planning has only finitely many states
    - » Thus, can make depth-first search complete by doing loop-checking

# Branching Factor of Forward Search



- Forward search can have a very large branching factor
  - ◆ E.g., many applicable actions that don't progress toward goal
- Why this is bad:
  - ◆ Deterministic implementations can waste time trying lots of irrelevant actions
- Need a good heuristic function and/or pruning procedure
  - ◆ See Section 4.5 (Domain-Specific State-Space Planning) and Part III (Heuristics and Control Strategies)

# Backward Search

- For forward search, we started at the initial state and computed state transitions
  - ◆ new state =  $\gamma(s, a)$
- For backward search, we start at the goal and compute inverse state transitions
  - ◆ new set of subgoals =  $\gamma^{-1}(g, a)$
- To define  $\gamma^{-1}(g, a)$ , must first define *relevance*:
  - ◆ An action  $a$  is relevant for a goal  $g$  if
    - »  $a$  makes at least one of  $g$ 's literals true
      - $g \cap \text{effects}(a) \neq \emptyset$
    - »  $a$  does not make any of  $g$ 's literals false
      - $g^+ \cap \text{effects}^-(a) = \emptyset$  and  $g^- \cap \text{effects}^+(a) = \emptyset$



# Inverse State Transitions

- If  $a$  is relevant for  $g$ , then
  - ◆  $\gamma^{-1}(g, a) = (g - \text{effects}(a)) \cup \text{precond}(a)$
- Otherwise  $\gamma^{-1}(g, a)$  is undefined
  
- Example: suppose that
  - ◆  $g = \{\text{on}(b1, b2), \text{on}(b2, b3)\}$
  - ◆  $a = \text{stack}(b1, b2)$
- What is  $\gamma^{-1}(g, a)$ ?

Backward-search( $O, s_0, g$ )

$\pi \leftarrow$  the empty plan

loop

if  $s_0$  satisfies  $g$  then return  $\pi$

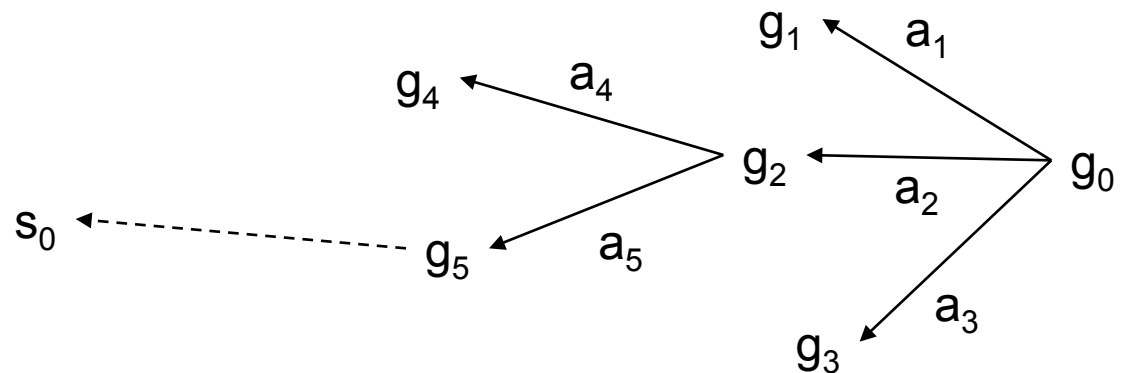
$A \leftarrow \{a \mid a \text{ is a ground instance of an operator in } O$   
and  $\gamma^{-1}(g, a)$  is defined}

if  $A = \emptyset$  then return failure

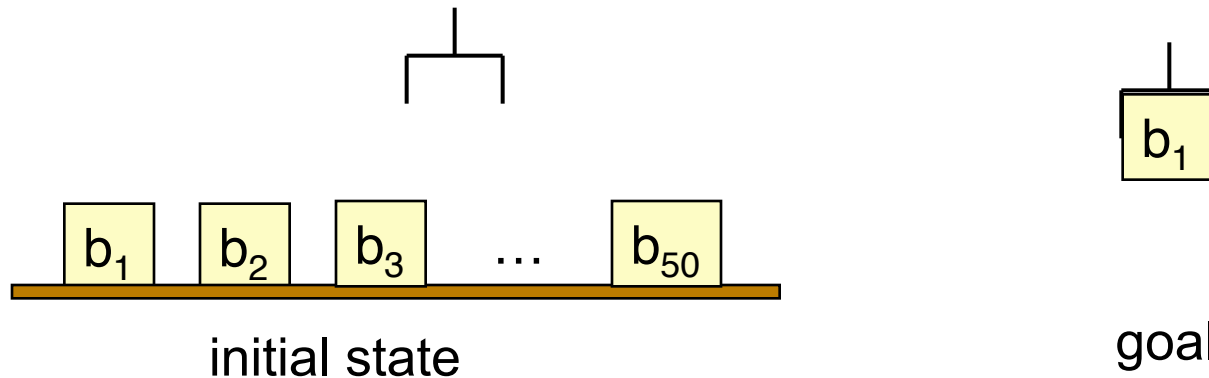
nondeterministically choose an action  $a \in A$

$\pi \leftarrow a.\pi$

$g \leftarrow \gamma^{-1}(g, a)$

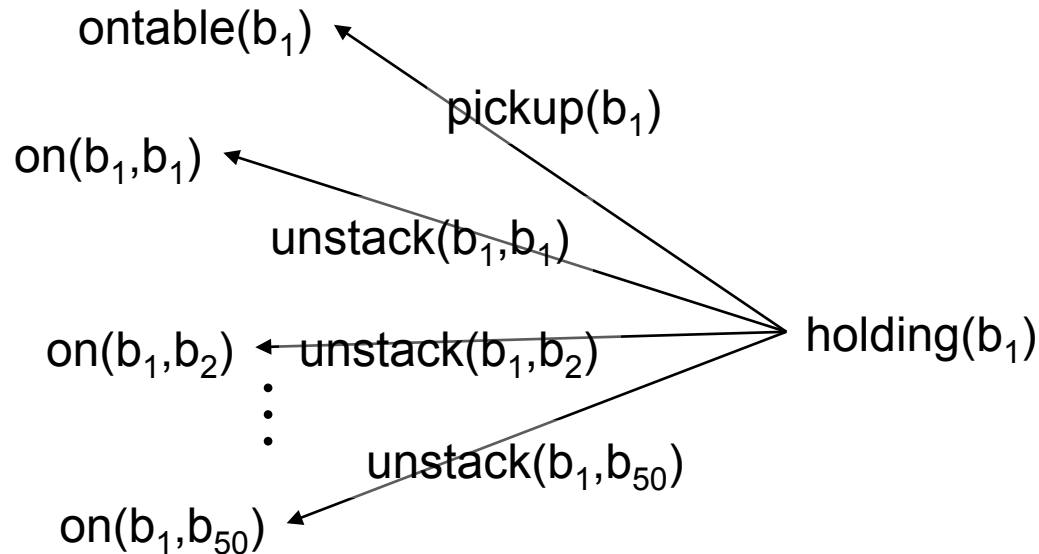


# Efficiency of Backward Search

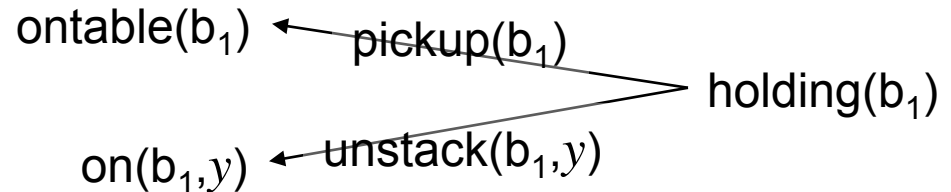


- Backward search can *also* have a very large branching factor
  - ◆ E.g., an operator  $o$  that is relevant for  $g$  may have many ground instances  $a_1, a_2, \dots, a_n$  such that each  $a_i$ 's input state might be unreachable from the initial state
- As before, deterministic implementations can waste lots of time trying all of them

# Lifting



- Can reduce the branching factor of backward search if we *partially* instantiate the operators
  - ◆ this is called *lifting*



# Lifted Backward Search

- More complicated than Backward-search
  - ◆ Have to keep track of what substitutions were performed
- But it has a much smaller branching factor

Lifted-backward-search( $O, s_0, g$ )

$\pi \leftarrow$  the empty plan

loop

if  $s_0$  satisfies  $g$  then return  $\pi$

$A \leftarrow \{(o, \theta) \mid o \text{ is a standardization of an operator in } O,$   
 $\theta \text{ is an mgu for an atom of } g \text{ and an atom of effects}^+(o),$   
 $\text{and } \gamma^{-1}(\theta(g), \theta(o)) \text{ is defined}\}$

if  $A = \emptyset$  then return failure

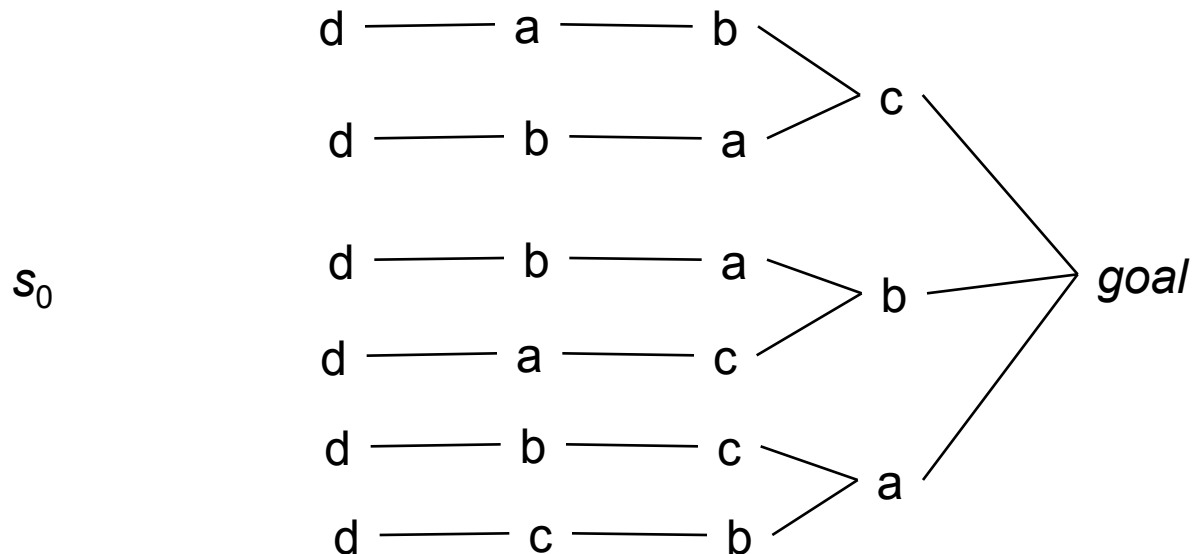
nondeterministically choose a pair  $(o, \theta) \in A$

$\pi \leftarrow$  the concatenation of  $\theta(o)$  and  $\theta(\pi)$

$g \leftarrow \gamma^{-1}(\theta(g), \theta(o))$

# The Search Space is Still Too Large

- Lifted-backward-search generates a smaller search space than Backward-search, but it still can be quite large
  - ◆ Suppose actions  $a$ ,  $b$ , and  $c$  are independent, action  $d$  must precede all of them, and there's no path from  $s_0$  to  $d$ 's input state
  - ◆ We'll try all possible orderings of  $a$ ,  $b$ , and  $c$  before realizing there is no solution
  - ◆ More about this in Chapter 5 (Plan-Space Planning)



# Pruning the Search Space

- I'll say a lot about this later, in Part III of the book
- For now, just two examples:
  - ◆ STRIPS
  - ◆ Block stacking

# STRIPS

- Basic idea: given a compound goal  $g = \{g_1, g_1, \dots\}$ , try to solve each  $g_i$  separately
  - ◆ Works if the goals are *serializable* (can be solved in some linear order)

$\pi \leftarrow$  the empty plan

do a modified backward search from  $g$ :

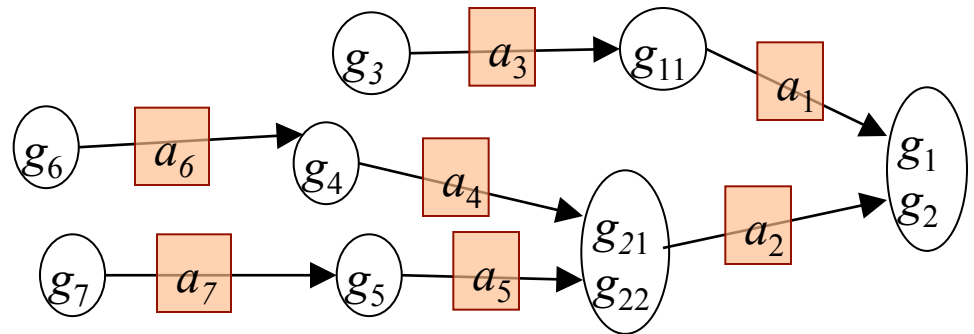
instead of  $\gamma^{-1}(s, a)$ , each new set of subgoals is just  $\text{precond}(a)$

whenever you find an action that's executable in the current state,

go forward on the current search path as far as possible,  
executing actions and appending them to  $\pi$

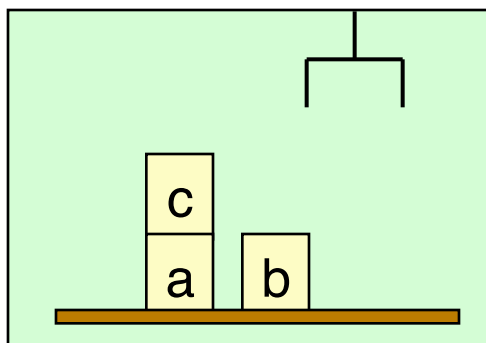
repeat until all goals are satisfied

$$\begin{aligned}\pi &= \langle \pi_1, \pi_2 \rangle \text{ or } \langle \pi_2, \pi_1 \rangle \\ \pi_2 &= \langle \pi_{11}, \pi_{12}, a_2 \rangle \text{ or } \langle \pi_{12}, \pi_{11}, a_2 \rangle \\ \pi_{21} &= \langle a_7, a_4 \rangle \\ \pi_{22} &= \langle a_7, a_5 \rangle\end{aligned}$$

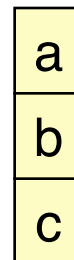




# The Sussman Anomaly



Initial state



goal

- On this problem, STRIPS can't produce an irredundant solution
  - ◆ Try it and see

# The Register Assignment Problem

- Interchange the values stored in two registers

- ◆ State-variable formulation:

- » registers  $r_1, r_2, r_3$

$s_0$ : {value( $r_1$ )=3, value( $r_2$ )=5, value( $r_3$ )=0}

$g$ : {value( $r_1$ )=5, value( $r_2$ )=3}

Operator:  $\text{assign}(r, v, r', v')$

precond: value( $r$ )= $v$ , value( $r'$ )= $v'$

effects: value( $r$ )= $v'$

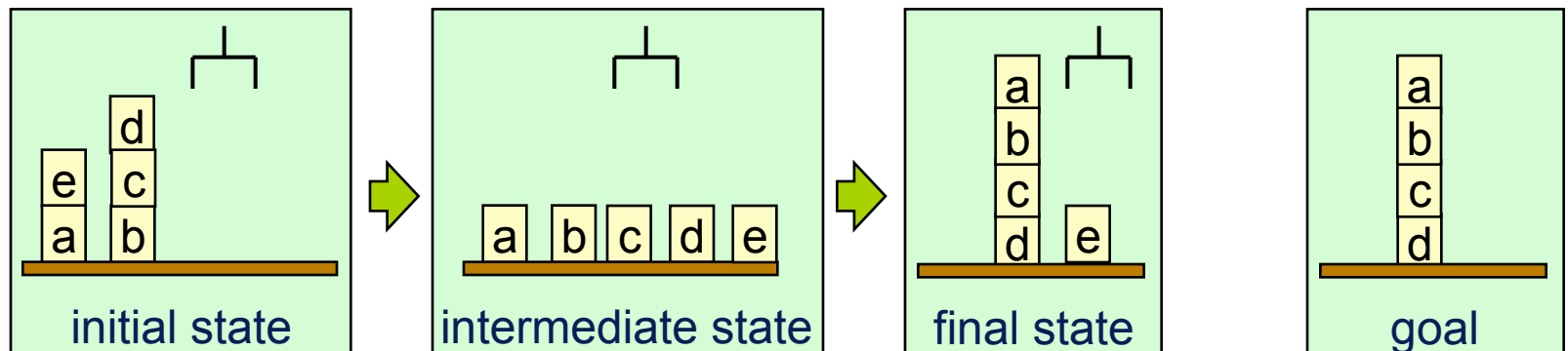
- STRIPS cannot solve this problem at all

# How to Handle Problems like These?

- Several ways:
  - ◆ Use a planning algorithm other than state-space search
    - » e.g., Chapters 5–8
  - ◆ Write a domain-specific algorithm
    - » Example: the blocks world

# Domain-Specific Knowledge

- A blocks-world planning problem  $P = (O, s_0, g)$  is solvable iff  $s_0$  and  $g$  satisfy some simple consistency conditions
  - ◆ no block can be on two other blocks at once, every block in  $g$  must also be in  $s_0$ , etc.
    - » Can check these in time  $O(n \log n)$
- If  $P$  is solvable, can easily construct a solution of length  $O(2m)$ , where  $m$  is the number of blocks
  - ◆ Move all blocks to the table, then build up stacks from the bottom
    - » Can do this in time  $O(n)$
- With additional domain-specific knowledge, can do even better (*next slide*)



# Block-Stacking Algorithm

- All of the possible situations in which a block  $x$  needs to be moved:
  - ◆  $s$  contains  $\text{ontable}(x)$  and  $g$  contains  $\text{on}(x,y)$  - e.g., a
  - ◆  $s$  contains  $\text{on}(x,y)$  and  $g$  contains  $\text{ontable}(x)$  - e.g., d
  - ◆  $s$  contains  $\text{on}(x,y)$  and  $g$  contains  $\text{on}(x,z)$  for some  $y \neq z$  - e.g., c
  - ◆  $s$  contains  $\text{on}(x,y)$  and  $y$  needs to be moved - e.g., e

**loop**

**if** there is a clear block  $x$  that needs to be moved

**and**  $x$  can be moved to a place where it won't need to be moved

**then** move  $x$  to that place

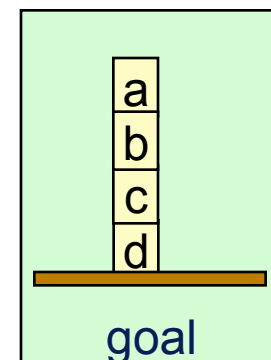
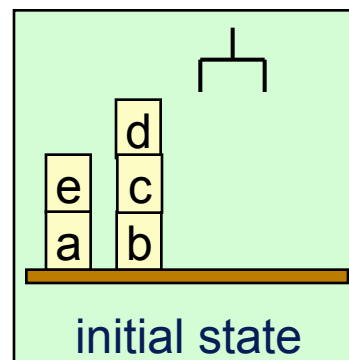
**else if** there's a clear block  $x$  that needs to be moved

**then** move  $x$  to the table

**else if** the goal is satisfied **then return** the plan

**else return** failure

**repeat**



# Properties of the Block-Stacking Algorithm

- Sound, complete, guaranteed to terminate
- Easily solves problems like the Sussman anomaly
- Runs in time  $O(n^3)$ 
  - ◆ Can be modified to run in time  $O(n)$
- Often finds optimal (shortest) solutions
- But sometimes only near-optimal (Exercise 4.22 in the book)
  - ◆ Recall that PLAN LENGTH for the blocks world is NP-complete