#### **Introduction to Game Theory**

#### **Review for the Final Exam**

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Nau: Game Theory 1

# **1. Introduction**

- Basic concepts:
  - > normal form, utilities/payoffs, pure strategies, mixed strategies
- How utilities relate to rational preferences (not in the book)
- Some classifications of games based on their payoffs
  - > Zero-sum
    - Roshambo, Matching Pennies
  - > Non-zero-sum
    - Chocolate Dilemma, Prisoner's Dilemma, Battle of the Sexes, Which Side of the Road?
  - Common-payoff
    - Which Side of the Road?
  - > Symmetric
    - all of the above except Battle of the Sexes

# 2. Analyzing Normal-Form Games

- I've discussed several solution concepts, and ways of finding them:
  - Pareto optimality
    - Prisoner's Dilemma, Which Side of the Road
  - best responses and Nash equilibria
    - Battle of the Sexes, Matching Pennies
    - finding Nash equilibria
  - real-world examples
    - soccer penalty kicks
    - road networks (Braess's Paradox)

# 3. More about Normal-Form Games

- maximin and minimax strategies, and the Minimax Theorem
  - Matching Pennies, Two-Finger Morra
- dominant strategies
  - Prisoner's Dilemma, Which Side of the Road, Matching Pennies
  - Elimination of dominated strategies
- rationalizability
  - the *p*-Beauty Contest
- correlated equilibrium
  - Battle of the Sexes
- trembling-hand perfect equilibria
- epsilon-Nash equilibria
- evolutionarily stable strategies
  - Hawk-Dove game

#### 4a. Extensive-Form Games

- Extensive-form games
  - relation to normal-form games
  - Nash equilibria
  - subgame-perfect equilibria
  - backward induction
    - The Centipede Game

# 4b. Game-Tree Search

- Two-player perfect-information zero-sum games
  - > the Minimax theorem applies
  - > perfect-info => only need to look at pure strategies
  - minimax game-tree search
    - minimax values, alpha-beta pruning
- In sufficiently complicated games, must compute approximations
  - limited search depth, static evaluation function
- In games that are even more complicated, further approximation is needed
  - Monte Carlo roll-outs

# 4c. Lookahead Pathology

- Probability of correct decision, critical nodes
  - examples (P-games and N-games)
- General results
  - Pathology is more likely when branching factor is high, granularity is small, local similarity is low
  - > Kalah, chess
  - Local pathologies

# **5. Imperfect-Information Games**

- Nodes partitioned into information sets
  - > Information set = {all the nodes you might be at}
- Behavioral strategies versus mixed strategies
  - > Different equilibria in general; same equilibria if there's perfect recall
- Sequential equilibria
  - > Like subgame-perfect equilibria, but with forests rather than trees
    - Example (in the homework) but no definition

• Monte Carlo game-tree generation, state aggregation

- example: Bridge programs
- Information-set search
  - compute a best response to opponent's strategy
  - paranoid and overconfident opponent models
  - results in kriegspiel, P-games, N-games, kalah
  - Brief discussion of poker

## **6a. Repeated Games**

- Finitely and infinitely repeated games
  - iterations, stage games
  - Roshambo, IPD, IPD with noise
- strategies for such games
- Differences between theoretical predictions and empirical results
- Examples:

> Roshambo

- Iterated Prisoner's Dilemma
- Noisy IPD

> Opponent modeling and noise filtering

### **6b Stochastic Games**

- Markov games
  - states, transition probabilities, reward functions, strategies, and equilibria
- Two-player zero-sum stochastic games

> expectiminimax

- Evolutionary simulation games
  - replicator and imitate-the-better dynamics
  - Iottery games, state-dependent risk preferences
- - Imitation dynamics
- Evolutionary stag hunt

# 7a. Incomplete-Information Games

- Regret, maximum regret, minimax regret
- Bayesian games
  - Didn't give a definition, but discussed necessary conditions
- Example of reducing an incomplete-information game to an imperfectinformation game
  - uncertainty about payoffs
- Auctions, and equilibrium analysis of them
  - English auction
  - -> The "dollar auction"
    - First-priced sealed-bid
    - > Dutch
    - Second-priced sealed-bid



# **8 Coalitional Games**

- Transferable utility
- Voting game example
- Classes of coalitional games
  - superadditive, additive, constant-sum, convex, simple, proper-simple, etc.
- Payoff sets, pre-imputation and imputation sets, Shapley value, etc.
- Core, stability