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Two examples of mechanism design

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Mechanism Design

The "mechanism" is how the game works
Design the game so that if the players act rationally,
they will produce a desired outcome

I don't know very much about this, but it's one of the most active areas of game-theory research right now

I'll give you two examples
(1) changing the rules of an auction
(2) adding a "mediator" to the Prisoner's Dilemma

Before I can talk about auctions, I need to talk about incomplete-information games

Incomplete-Information Games

A fact is *common knowledge* if (1) every player knows it
(2) every player knows (1)
(3) every player knows (2)
. . .

A player's *private information* is any info he/she has that's not common knowledge among all the players in the game.

Incomplete-information game: at the first point in time where the players can begin to plan their moves, some players already have private information about the game that the other players do not know.

A *Bayesian Game* is a model of an incomplete-information game

Bayesian Games

Let N be a set of players. A Bayesian game is a tuple

$$\Gamma^b = (N, (C_i)_{i \in N}, (T_i)_{i \in N}, (p_i)_{i \in N}, (u_i)_{i \in N}),$$

where for each player $i \in N$,

C_i is i's set of possible actions,

T_i is i's set of possible *types* (possible values for i's private information),

p_i and u_i are as defined below.

Let $C = \times_{i \in N} C_i$ and $T = \times_{i \in N} T_i$

For each i, $\{$all possible combinations of types for the other players$\}$ is

$T_{-i} = \times_{j \in N-i} T_j$

For each $t_i \in T_i$, p_i specifies a probability distribution $p_i(.|t_i)$ over T_{-i}

p_i is what i will believe about the other player's types if i's type is t_i

Given an action profile $c \in \times_{i \in N}$ and a type profile $t \in \times_{i \in N}$,

$u_i(c, t)$ is the payoff for player i

Bayesian equilibrium

A Bayesian equilibrium is a Nash equilibrium over the agents' types

Each player maximizes his/her own expected utility when he/she knows his/her type but doesn't know the other agents' types

Auction with independent private values

Consider a sealed-bid auction for some object. Each player i knows how much the object would be worth to him/her, namely v_i (private information)

For every $j \neq i$, suppose i thinks player j's value v_j is a random variable from a probability distribution F over the interval $[0, M]$.

Let $b = (b_1, \ldots, b_n)$ and $v = (v_1, \ldots, v_n)$ be the profiles of bids and values. Then the playoff to player i is

$u_i(b, v) = v_i - b_i$ if i's bid is highest

$u_i(b, v) = 0$ otherwise

It's possible to show there is a differentiable and increasing function β such that the following profile is a Bayesian equilibrium:

each player's bid is $b_i = \beta(v_i)$

Auction with common value

Object has the same value v to each bidder (but different bidders may have different *estimates* of what v is)

Example: $v = A_0x_0 + A_1x_1 + A_2x_2$, where
A_0, A_1, A_2 are nonnegative constants that are commonly known
x_0, x_1, x_2 are uniformly distributed over [0,1]

player 1 knows x_0 and x_1 but not x_2
player 2 knows x_0 and x_2 but not x_1

This game has a unique Bayesian equilibrium in which
player 1's bid is $A_0x_0 + 0.5(A_1 + A_2)x_1$
player 2's bid is $A_0x_0 + 0.5(A_1 + A_2)x_2$

Example

Suppose $A_0 = A_1 = A_2 = 100$, $x_0 = 0$, $x_1 = 0.01$

player 1's estimate of v's value is

$$E(v) = A_0 x_0 + A_1 x_1 + A_2 E(x_2) = 100 \cdot 0 + 100 \cdot 0.01 + 100 E(x_2)$$
$$= 0 + 1 + 50 = 51$$

But player 1's bid is

$$A_0 x_0 + 0.5(A_1 + A_2) x_1 = 100 \cdot 0 + 0.5(100 + 100) 0.01 = 1$$

Why so low? Why not bid 50 instead?

Example

Suppose $A_0 = A_1 = A_2 = 100$, $x_0 = 0$, $x_1 = 0.01$

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But player 1's bid is

$$A_0 x_0 + 0.5(A_1 + A_2)x_1 = 100 \cdot 0 + 0.5(100 + 100)0.01 = 1$$

Why so low? Why not bid 50 instead?

Because player 1's estimate of v's value *given that player 1 bids 50 and wins* is only 38

More generally, for every number $b > 1$,
player 1's estimate of v's value *given that player 1 bids b and wins* is $v < b$

Vickrey auction

Sell the object to the highest bidder, at the price that was bid by the 2nd-highest bidder

In this case, the equilibrium strategy is for each player to bid what he/she things the object is actually worth

Mediators

Example: the Prisoner's Dilemma

		P_2	
		C	D
P_1	C	$3, 3$	$0, 5$
	D	$5, 0$	$1, 1$

Dominant strategy equilibrium: (D,D)

Mediator: a trustworthy agent with whom you can make an agreement so that the mediator makes your moves for you

Suppose the mediator agrees to make the following moves:

(1) if you use the mediator and the other player doesn't then the mediator will use D for your move

(2) if both you and the other player use the mediator, then the mediator will use C for both of you

Mediators

		P_2		
		C	D	M
P_1	C	$3,3$	$0,5$	$0,5$
	D	$5,0$	$1,1$	$1,1$
	M	$5,0$	$1,1$	$3,3$

(D,D) is no longer a dominant strategy equilibrium

(M,M) is, instead