A methodology for systematic generation and evaluation of alternative operation plans*

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Abstract

This chapter describes a methodology for analyzing some of the manufacturability aspects of machined parts during the design stage of the product development cycle, so that problems related to machining can be recognized and corrected while the product is being designed. Starting with the CAD design for a proposed part, our basic approach is to systematically generate alternative operation plans for machining the part, evaluating the capabilities of each operation plan to see which one best balances the need for efficient manufacturing against the need for a quality product. We anticipate that the information provided by this analysis will be useful both to provide information to the manufacturing engineer about alternative ways in which the part might be machined, and also to give feedback to the designer about problems that might arise with the machining.

1. INTRODUCTION

One of the missing links between CAD and CAM is the virtual absence of any systematic methodology for generating and evaluating the alternative ways to manufacture a proposed design. Most integrated CAD/CAM systems try to generate a single process plan for a given design—but in general, there may be several alternative ways to manufacture the design. How easy it is to manufacture—or whether it is even possible to manufacture it at all—may depend on which alternative is chosen. Thus, these alternatives should be generated and examined, to determine how well each one balances the need for a quality product against the need for efficient manufacturing.

In this chapter we describe a methodology for systematically generating and evaluating alternative operation plans for machined parts. Our approach involves representing

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the design as a collection of *machining features*, volumetric features that correspond to machining operations. In general, there may be several alternative representations of the design as different collections of machinable features, corresponding to different ways to machine the part. The basic idea is to generate alternative interpretations of the part as collections of machinable features, map these interpretations into operation plans, and evaluate the manufacturability of each operation plan. More specifically, our approach involves the following steps:

1. Build the set of all potential machining features $\mathcal{F}$ by identifying various features which can be used to create the part $P$ from the stock $S$, as described in Section 4. Each feature in $\mathcal{F}$ represents a different possible machining operation which can be used to create various surfaces of the part.

2. Do the following steps repeatedly, until every promising feature-based model (FBM) for $P$ has been examined:
   
   (a) Generate a promising FBM $F$ from the feature set $\mathcal{F}$, as described in Section 5. As described in Section 3, an FBM is basically a set of machining features that contains no redundant features and is sufficient to create the part $P$. We consider an FBM unpromising if it is not expected to result in any operation plans better than the ones which has already been examined.

   (b) Do the following steps repeatedly, until every promising operation plan resulting from $F$ has been examined:
      
      i. Generate a promising operation plan $O$ from $F$, as described in Section 6. $O$ represents a partially ordered set of machining operations. We consider an operation plan to be unpromising if it violates any common machining practices.

      ii. Estimate the achievable machining accuracy of the operation plan $O$, as described in Section 7.1. If the operation plan $O$ cannot produce the required design tolerances and surface finishes, then discard it and go to Step i.

      iii. Estimate the production time and cost associated with operation plan $O$, as described in Section 7.2.

3. If no promising operation plans were found during the above steps, then exit with failure. Otherwise exit with success, returning the operation plan that represents the best tradeoff among quality, cost, and time, as described in Section 7.3.

The results of such an analysis can potentially be used for two purposes: (1) to give the production engineer information about what processes and process parameters are most desirable over the various ways in which the part might be machined; and (2) to give the product designer a better understanding of whether and how the design might be changed to improve its manufacturability.
2. RELATED WORK

Feature-based approaches have been very popular in a variety of CAD/CAM implementations, but different people have used the term to mean different things [27, 9, 14, 6]. Significant amounts of work have been directed towards defining sets of form features to serve as a communication medium between design and manufacturing—but at present, most researchers are convinced that a single set of features cannot satisfy the requirements of both of these domains. The recent trend seems to be toward defining sets of features with specific application domains in mind (such as machining, assembly, inspection, etc.). For the machining domain, most researchers agree that volumetric features are preferable to surface features, although certain additional information about the surfaces is needed (for example, to determine accessibility and tool approach directions).

2.1 Recognizing Machining Features

There are three primary approaches for obtaining features from a CAD model. In human-supervised feature recognition, a human user examines an existing CAD model to determine what the manufacturing features are [1]. In automatic feature recognition, the same feature recognition task is performed by a computer system [4, 29, 25, 5, 11, 23]. In design by features, the designer specifies the initial CAD model in terms of various form features which translate directly into the relevant manufacturing features [26, 28, 10]. Many examples exist of each of these approaches. However, these approaches typically produce a single set of features describing the CAD model, rather than several alternative interpretations of the model.

2.2 Generating Alternatives

Hummel [9] and Mantyla [16] present examples of multiple feature representations of the same object. However, these papers do not describe a system or methodology for generating multiple feature models.

Hayes’s MACHINIST system [8] can identify certain cases in which one feature needs to be made before another. However, its representation of features is not adequate for all aspects of process planning. For example, if it decides that some hole needs to be made before some slot, it does not automatically update the dimensions of the hole or the slot—information which would be needed for process selection.

The AMPS process planning system [2] includes a step called “feature refinement,” which involves combining a set of features into a more complex feature if it appears that this will optimize the plan, or splitting a feature that cannot be machined into two or more features that can (hopefully) be machined. The techniques for deciding when to combine or split features are heuristic in nature, so from the author’s description it is not always clear when alternative interpretations will be produced.

Vandenbrande [29] has developed a system that combines techniques from artificial intelligence and solid modeling. It uses hints or clues to identify potential features in the boundary representation of the part. It is capable of identifying interacting features (e.g., two intersecting slots), and produces alternative feature interpretations in certain cases.

The first systematic work on generation of alternative interpretations was done by Karinthi and Nau [12, 13]. They describe an approach for producing alternative inter-
pretations of the same object as different collections of machining features as the result of algebraic operations on the features, and a system for generating alternative interpretations by performing these algebraic operations. This system works with abstract volumetric features. There is no direct relation between these features and machining operations. Therefore some of the interpretations generated by this approach are not feasible from the machining point of view. In this approach a set of algebraic operators (such as maximal extension, truncation etc.) has been used to generate new interpretations of the part. But this set of operators is not sufficient to generate all interpretations of the part. Moreover, many times the resulting features do not belong to any of the feature classes. Some of the feature interactions may also result in partial ordering among features, which is an important issue from a machining point of view—but this work does not deal with time orderings among the features.

2.3 Evaluating Operation Plans

Because of the need for quality assurance on the shop floor, extensive research has been done on different aspects of evaluation of operation plans. Much of the data relevant for machining operation planning is available in machining data handbooks such as [15]. In addition, mechanistic models have been developed to provide quantitative mappings between machining parameters (such as cutting speed, feed, and depth of cut), to the performance measures of interest (such as surface finish and dimensional accuracy) [7, 2, 35, 36, 30]. Research on machining economics has produced quantitative models for evaluating times and costs related to machining operations [37, 32, 31], and optimization techniques have been applied to these quantitative models to seek the machining parameters which minimize the variable cost, or maximize the production rate and profit rate associated with machining operations.

3. DEFINITIONS

3.1 Geometric Solids and Machined Parts

For our purposes, a solid is any regular, semi-analytic subset of three-dimensional Euclidean space. If \( R \) is any solid, then \( b(R) \) is the boundary of \( R \), and \( \iota(R) \) is the interior of \( R \). Note that \( R = \iota(R) \cup b(R) \) and that \( \iota(R) \cap b(R) = \emptyset \). If \( R \) and \( R' \) are solids, then \( R \cap^* R' \) is the regularized intersection of \( R \) and \( R' \), i.e., the closure of \( \iota(R) \cap \iota(R') \). Similarly, \( R \cup^* R' \) and \( R -^* R' \) are the regularized union and regularized difference, respectively. For further details on these and other related concepts, see [24].

A machined part (or just a part) is the finished component \( P \) to be produced as a result of a sequence of machining operations on a piece of stock \( S \), which is the raw material from which the part is to be machined. The delta volume is the volume to be machined, i.e., \( \Delta = S -^* P \). The workpiece is the intermediate part \( W \) produced by performing one or more of the machining operations in the sequence. We will represent \( P, S, \) and \( W \) as geometric solids.

3.2 Machining Features

In a machining operation, material is removed by relative motion between the cutting tool and the workpiece. The cutting tool is mounted on a large machine tool, and the
total volume occupied by the cutting tool and the machine tool is quite large. But we will only be interested in some small portion of this total volume, namely the portion that actually gets close to the workpiece. We will call this portion the tool volume, and we will denote it by $T$. Fig. 1(a) shows a drilling tool. To perform a cutting operation, the tool volume $T$ is given a relative cutting motion with respect to the workpiece. This cutting motion may either be imparted to the tool (examples include various milling operations) or the workpiece (examples include various lathe operations). Most of the time this relative cutting motion is either linear (operations such as shaping, planning, broaching) or rotational (operations such as turning, drilling, boring, milling). We represent this motion as a sweep $s_v$ which is either linear or rotational. Let $T_v$ denote the solid generated by applying sweep $s_v$ to the solid $T$. For the purpose of locating the tool, we choose a particular point $p_d$ of $T_v$ as a datum point. Fig. 1(b) shows $T_v$ and $p_d$ for drilling operations.

For our purposes, a machining feature is the portion of the workpiece affected by a machining operation. However, we will need to know not just the volume of material which the feature can remove from the workpiece, but also what kind of machining operation we are performing, and how we access the workpiece in order to perform the operation. More formally, a machining feature is a triple $f = (\text{rem}(f), \text{acc}(f), \text{class}(f))$, where $\text{rem}(f)$, $\text{acc}(f)$, and $\text{class}(f)$ are as defined below:

- To perform the machining operation, one sweeps the volume $T_v$ along some trajectory $t$. The trajectory $t$ is feasible only if sweeping $T_v$ along $t$ does not cause
interference problems between the non-cutting surface of $T_v$ and the workpiece. Fig. 1(b) shows an example of a feasible tool trajectory for drilling. If $t$ is feasible, then the solid created by sweeping $T_v$ is $S_{sv} = \{ (p - p_d) + q : p \in T \text{ and } q \in t \}$, as shown in Fig. 1(c). However, only a portion of $S_{sv}$ actually corresponds to the volume that can be removed by the machining feature.

Let the approach surface $\pi$ be a surface touching solid $T_v$ and containing $T_v$ to one of its sides. This surface is either a plane or a cylinder depending upon the machining operation. For drilling operations this surface is planar as shown in Fig. 1(b). The side containing $T_v$ is called accessibility side. The other side is called removal side. The approach face of $f$ is defined as $a(f) = \pi \cap S_{sv}$. The removal volume of $f$ is the solid $\text{rem}(f)$ consisting of all points in $S_{sv}$ that are on the removal side of $\pi$. The effective removal volume of $f$ is the intersection between $\text{rem}(f)$ and the delta volume; i.e., it is the solid $\text{rem}(f) \cap^* \Delta$.

- The accessibility volume for $f$ is the solid $\text{acc}(f)$ consisting of all points in $S_{sv}$ that are on the accessibility side of $\pi$.

- The feature $f$ will be an instance of some feature class $\phi$. The feature class $\phi$ is a parameterized set of machining features corresponding to some machining operation $o$, and it is characterized by the shape and trajectory of the cutting tool used to perform the operation $o$. If $f$ is a feature in $\phi$, then the class of $f$ is $\text{class}(f) = \phi$, and the machining operation for $f$ is $\text{op}(f) = o$.\footnote{In order to create a feature, sometimes we will need several machining operations: a roughing operation followed by one or more finishing operations. In this chapter, we do not handle such cases; instead, we assume that each feature $f$ can be made using a single machining operation $\text{op}(f)$. This restriction significantly limits the scope of the current work, but we intend to remove this restriction in our future work. We believe that it will be relatively straightforward to do so by using techniques similar to those we employed in our previous work on process selection for roughing and finishing operations [17].} If $f$ is an instance of $\phi$, then the $f$’s parameters in $\phi$ are the specific set of parameter values for $\phi$ that yield $f$. Below are two examples:
If we are interested in drilling holes, then we may define \( \phi_d \) to be the set of all features that can be created by sweeping a drilling tool of diameter \( d \) along a linear trajectory starting at the datum point \( p_d \) and going in along some unit vector \( \vec{v} \) for some distance \( l \). Thus, we can specify a particular feature in \( \phi_d \) by giving specific values for \( p_d, \vec{v}, d, \) and \( l \).

- If we are interested in making milled pockets, slots or faces, then we may define \( \phi_m \) to be the set of all features that can be created by sweeping a milling tool of radius \( r \) in plane, whose parameters are the starting point \( p_d \), the depth \( l \), the edge loop \( e \), and the unit orientation vector \( \vec{v} \). Thus, we can specify a particular feature in \( \phi_m \) by giving specific values for \( p_d, \vec{v}, e, \) and \( l \).

Fig. 2 gives some examples of machining features.

A feature \( f \) is accessible in a workpiece \( W \) if the following conditions are satisfied:

1. \( f \)'s accessibility volume does not intersect the workpiece, i.e., \( \text{acc}(f) \cap^* W = \emptyset \).

2. If \( f \)'s class is drilling, then to ensure proper machining, the hole’s entry face should be a planar surface perpendicular to the hole’s axis (no similar condition is needed for milling).

### 3.3 Feature-Based Models

Depending upon available manufacturing facilities, we will have some fixed finite set of feature classes \( \Phi = \{\phi_1, \ldots, \phi_n\} \), and for each part that we want to manufacture, we will be interested in describing the part in terms of features from \( \Phi \). Suppose we are given a part \( P \) and stock \( S \). A feature-based model (or FBM) of \( P \) and \( S \) is any set of features \( F \) having the following properties:

1. Each \( f \in F \) is an instance of some feature class in \( \Phi \).
2. If we subtract the features in \( F \) from \( S \), we get \( P \); i.e., \( S - \bigcup_{f \in F} \text{rem}(f) = P \).
3. No feature in \( F \) is redundant, i.e., for every feature \( f \in F \), \( S - \bigcup_{g \in F - \{f\}} \text{rem}(g) \neq P \).
Figure 4: An example of a datum-dependency precedence constraint. In this case, s1 should be made before h.

For example, Fig. 3 shows a simple part and two FBMs for it. Intuitively, an FBM is an interpretation of the delta volume as a set of machining features.

Let f and f' be any two distinct features in some FBM. Then f and f' intersect each other if \( \text{rem}(f) \cap^* \text{rem}(f') \neq \emptyset \).

3.4 Precedence Constraints and Operation Plans

Due to accessibility [18], setup [8] and other types of interactions [20] among the features in an FBM F, the features of F cannot be machined in any arbitrary order. Instead, these interactions will introduce precedence constraints requiring that some features of F be machined before or after other features.

Let F be an FBM, and let f and f' be any two features in F. We will be interested in the following two types of precedence constraints among f and f':

1. **Accessibility precedence constraint.** If \( \text{acc}(f) \cap^* \text{rem}(f') \neq \emptyset \), then this means that the cutting tool approaches f through the volume occupied by f', and thus f' must be machined before f. An example is shown in Fig. 3(c), in which the pocket must be machined before the hole.

2. **Minimality precedence constraint.** Suppose that machining f' before f would allow us to machine rem(f) using a smaller feature g of the same class as f, then we constrain f to be machined before f' (for otherwise, we would be machining g rather than f). An example is shown in Fig. 3(d), in which we would constrain the hole to be machined before the pocket.

3. **Datum-dependency precedence constraint.** If feature f creates the datum surface for feature g, then f should be machined before g. An example is shown in Fig. 4.

If the precedence constraints contradict each other (i.e., if there is no total ordering consistent with them), then we consider F to be unmachinable. Otherwise, the precedence constraints will induce a partial order \( \prec \) on the features of F (i.e., \( f \prec f' \) if f must be machined before f')—or equivalently, they will induce a partial order on the machining
Table 1

<table>
<thead>
<tr>
<th>Surfaces created by drilling and milling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bottom Side</td>
</tr>
<tr>
<td>drilling conical (concave)</td>
</tr>
<tr>
<td>milling planar</td>
</tr>
<tr>
<td>Side</td>
</tr>
<tr>
<td>cylindrical (concave)</td>
</tr>
<tr>
<td>cylindrical or planar</td>
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operations corresponding to the features in $F$ (i.e., $\text{op}(f) \prec \text{op}(f')$ iff $f \prec f'$). In this case, the operation plan for $F$ consists of the set of machining operations $O$ along with the partial ordering $\prec$. Note that every total ordering $\{o_1, o_2, \ldots, o_k\}$ of $O$ that is consistent with the operation plan will satisfy the following conditions:

For each $i$, let $f_i$ be the feature corresponding to $o_i$. Let $W_0 = S$, and $W_i = W_{i-1} - *\text{rem}(f_i)$ for all $i > 0$. Then

**Condition 1:** for all $i > 0$, $f_i$ is accessible in $W_{i-1}$, i.e., $\text{acc}(f_i) \cap *W_{i-1} = \emptyset$.

**Condition 2:** each $f_i$ is the smallest feature in its class that can be used to produce $W_i$ from $W_{i-1}$; i.e., there is no feature $f \in \text{class}(f_i)$ such that $\text{rem}(f) \subset \text{rem}(f_i)$ and $W_{i-1} - *\text{rem}(f_i) = W_{i-1} - *\text{rem}(f)$.

4. **IDENTIFYING MACHINING FEATURES**

Given solids representing the part $P$ and the stock $S$, and a set of feature classes $\Phi$, we are interested in finding the set $\mathcal{F}$ of all features from $\Phi$ that correspond to machining operations that can be used to create $P$. Each machining feature is capable of creating certain types of surfaces. For example, Table 1 presents the types of surfaces that can be created by drilling (shown in Fig. 2(a)) and milling (shown in Fig. 2(b)).

In our approach, we consider all the part surfaces that need to be created, and try to identify features (i.e., instances of feature classes) which are capable of creating those surfaces. The basic idea behind our approach is given below:

Let $U = b(P) - *b(S)$ be the set of all faces of $P$ that are not faces of $S$. These are the faces of $P$ that will need to be machined. For each face $u \in U$, do the following:

For each feature class $\phi \in \Phi$, add to $\mathcal{F}$ every feature $f \in \phi$ that has the following properties:

1. $f$ can create $u$ (i.e., $u$ is a subface of some face of $f$), and $f$ does not intersect the part (i.e., $P \cap *\text{rem}(f) = \emptyset$),
2. for every feature $g \in \phi$ that has property 1 and contains $f$, $f$ and $g$ have the same effective removal volume.
3. for every feature $g \in \phi$ that has property 1 and is contained by $f$, $g$ has a smaller effective removal volume.
As a specific instance of this approach, we have developed an algorithm for identifying machining features from a given portion of the boundary of the feature. For the details of the algorithm, readers are referred to [23, 22]. This algorithm handles a large variety of features that correspond to drilling and milling operations, and it is provably complete, even if the features intersect with each other in arbitrarily complex ways.

The primary limitation of this algorithm is that it is designed only to handle linearly swept features (i.e., holes, slots, pockets etc.). However, our definitions of drilling and milling features are more general than the definitions used in a number of feature recognition systems; for example, milled pockets may be complicated swept contours that include corner radii, islands and other characteristics, in order to realistically describe a non-trivial set of mechanical parts. For example, the algorithm can handle each of the objects shown in Fig. 5.

As an example, consider the part shown in Fig. 6. Let us assume that this part will be machined from a rectangular stock made of plain carbon steel (100 BHN), measuring 80mm $\times$ 80mm $\times$ 55mm. Suppose that the only available feature classes are the class of all drilling features and the class of all milling features. Then Fig. 7 shows the features identified by our algorithm.

5. GENERATING FEATURE-BASED MODELS

After finding the set of features $\mathcal{F}$, the next step is to use these features to generate FBMs for $P$ and $S$. Since each FBM is basically an irredundant set cover for the set $\mathcal{F}$, we will generate irredundant FBMs using irredundant-set-covering techniques [21, 19], and use pruning heuristics to discard unpromising FBMs.

Discarding Unpromising FBMs. Let $F$ be an FBM, and let $L_s(F)$ be the cardinality of the set $\{\bar{v}(f) : f \in F\}$, where $\bar{v}(f)$ is the unit orientation vector for feature $f$. Then
Figure 6: An example part.
\[ \Phi = \{ \text{drilling features, milling features} \} \]

\[ \mathcal{F} = \{ s_1, s_1', s_2, s_3, s_4, s_4', s_5, s_5', h, h' \} \]

Figure 7: Features identified by our algorithm for the part shown in Fig. 6.
Figure 8: FBMs generated by our algorithm.

$L_s(F)$ is the number of different directions of approach needed in order to machine $F$, and (except on 5-axis machines or special purpose fixtures) is a lower bound on the number of setups needed to machine $F$. Similarly, let $L_{tc}(F)$ be the cardinality of the set $\{(\text{tool}(f), \vec{v}(f)) : f \in F\}$, where $\text{tool}(f)$ is the tool associated with feature $f$. Then $L_{tc}(F)$ is a lower bound on the number of tool changes needed in order to machine $F$.

If two FBMs $F$ and $F'$ have same sets of removal volumes but different sets of accessibility volumes, then the expected machining accuracy of $F$ and $F'$ is same, but the number of setups and/or tool changes might be different. In this case, we consider $F'$ unpromising if either of the following conditions is satisfied:

- $L_s(F) \leq L_s(F')$ and $L_{tc}(F) < L_{tc}(F')$, i.e., $F$ is believed to require no more setups than $F'$, and fewer tool changes;

- $L_{tc}(F) \leq L_{tc}(F')$ and $L_s(F) < L_s(F')$, i.e., $F$ is believed to require no more tool changes than $F'$, and fewer setups.

As an example, Fig. 8 shows two of the FBMs generated from the features shown in Fig. 7.
6. GENERATING OPERATION PLANS

After generating FBM$s, the next step is to generate the associated machining operations along with their partial orderings. Given an FBM $F$, we generate operation plans from $F$ as follows:

1. $O = \emptyset$. (O will eventually be the set of operation plans returned in Step 3.)
2. For every partial ordering $<$ on $F$ that totally orders intersecting features and leaves non-intersecting features unordered,\(^2\) do:
   (a) Let $f_1, \ldots, f_n$ be any total ordering of $F$ that is consistent with $<$.\(^3\) As described below, trim each $f_i$ with respect to $W_i = S^{-*} (\text{rem}(f_1) \cup^* \ldots \cup^* \text{rem}(f_{i-1}))$, producing a new trimmed feature $g_i$. If $g_i$ is not accessible in $W_i$, then discard $<$ and skip Steps (b) and (c). Otherwise, let $G$ be the FBM consisting of $g_1, \ldots, g_n$.
   (b) Let $\rightarrow$ be the partial ordering on $G$ that is defined as follows:
      i. $g \rightarrow g'$ for each pair of features $g, g' \in G$ such that $g$ must be machined in order to make $g'$ accessible (i.e., $\text{rem}(g) \cap^* \text{acc}(g') \neq \emptyset$);
      ii. $g \rightarrow g'$ for each pair of features $g, g' \in G$ such that machining $g'$ before $g$ would allow us to machine $\text{rem}(g)$ using a smaller feature $h$ of the same class as $g$ (i.e., there is a feature $h \in \text{class}(g)$ such that $\text{rem}(h) \subset \text{rem}(g)$ and $S^{-*} (\text{rem}(g') \cup^* \text{rem}(h)) = S^{-*} (\text{rem}(g') \cup^* \text{rem}(g))$).
      iii. $g \rightarrow g'$ for each pair of features $g, g' \in G$ such that $g$ creates the datum surface for $g'$.
   (c) If $\rightarrow$ is a consistent partial ordering (which can easily be verified using a topological sorting procedure \[3\]), then $(G, \rightarrow)$ is an operation plan, so add it to $O$ and select the associated cutting parameters. Otherwise, $G$ is not machinable, so discard it.
3. Return $O$.

If $f$ is a feature and $R$ is a solid, then trimming $f$ with respect to $R$ involves the following two steps:

\(^2\)What we mean by this is the following. Let $\{(f_1, f'_1), (f_2, f'_2), \ldots, (f_n, f'_n)\}$ be all pairs of intersecting features in $F$, and for each $i$, let $C_i$ be the pair of partial ordering constraints $\{(f_i < f'_i), (f'_i < f_i)\}$. Then every consistent set of partial ordering constraints that can be found in the Cartesian product $C_1 \times \ldots \times C_n$ gives us a possibility for the partial ordering $\prec$. In the worst case, this could be a very large number of partial orders—but we believe that this worst case is quite unlikely to occur. In most cases, all sets of intersecting features will be quite small, and thus the number of partial orders should not normally be very large.

\(^3\)Such a total ordering can easily be generated using topological sorting \[3\]. This total ordering is not unique, but since $\prec$ totally orders intersecting features, we can prove that we will get exactly the same operation plan regardless of which total ordering is produced by the topological sorting algorithm. The only purpose of this total ordering is so that we can trim the features; once we have trimmed them we discard the total ordering.
1. First, shorten \( f \) by eliminating (as much as possible) those portions of \( \text{rem}(f) \) that are outside \( R \) and finding a new datum point \( p_d \).

2. The removal volume \( \text{rem}(f) \) is a swept volume produced by sweeping the cutting-tool volume \( T_v \) along a trajectory \( t \) that starts at the datum point \( p_d \). If the trajectory \( t \) can be shortened without changing the datum point \( p_d \) or the volume removed from \( R \) by \( f \), then trim \( f \) by shortening \( t \).

As an example, consider FBM 1 of Fig. 8. If the hole \( h \) is machined before the face \( s4 \), then \( h \)'s entry face will not be perpendicular to its axis and will pose an accessibility problem (as described at the end of Section 3.2). Therefore, the above procedure will generate no operation plan in which \( s4 \prec h \). Fig. 9 shows two different operation plans produced from FBM 1.

**Identifying Unpromising Operation Plans.** We will consider \( O \) to be unpromising if it contains features whose dimensions and tolerances appear unreasonable. Examples include the following: a hole-drilling operation having too large a length-to-diameter ratio; a recess-boring operation having too large a ratio of outer diameter to inner diameter; two concentric hole-drilling operations with tight concentricity tolerance and opposite approach directions. All three of these examples are illustrated in Fig. 10.

7. **OPERATION PLAN EVALUATION**

7.1 **Estimating Achievable Machining Tolerances**

Each machining operation creates a feature which has certain geometric variations compared to its nominal geometry. Designers normally give design tolerance specifications on the nominal geometry, to specify how large these variations are allowed to be. To verify whether or not a given operation plan will produce the desired design tolerances, we want to estimate what tolerances the operations can achieve.

To get the most accurate results, the best technique is to construct a mathematical model of the machining process. To date, we have done this for turning and boring—and our methodology can easily be extended to model all machining processes involving single-point cutting tools. By modeling the relative motion of the workpiece and the cutting tool, we produce models of topography resulting from the machining process—and from these models, we calculate the achievable tolerances and surface finishes produced by the machining process. Our models take into account the following factors:

1. The machining system parameters, such as the feed rate, cutting speed, depth of cut, and structural dynamics [38, 33, 34, 15, 18].

2. The natural and external variations in the machining process. For example, variations in hardness in the material being machined cause random vibration, which is one of the factors affecting the surface quality [35, 36, 18].

To model these factors, we use a combination of deterministic, statistical, and empirical techniques [35, 36, 38, 33, 34, 18].
Figure 9: Generating Operation Plans from FBM 1

Figure 10: Examples of features which lead to unpromising operation plans.
Machining processes that do not involve single-point cutting tools are complex. Mathematical models to describe drilling, milling, and grinding processes can be found in the relevant literature. In our approach, empirical models are also used to estimate machining tolerances. The approach is similar to tolerance charting. For the sake of brevity, we omit the details.

Example. For the part shown in Fig. 6, Fig. 11 shows two operation plans $O_a$ and $O_b$. $O_a$ was generated from FBM 1 (it is the rightmost of the plans shown in Fig. 9), and $O_b$ was generated from FBM 2. The details of these two plans are presented in the Appendix. As we discuss later in Section 7.2, $O_a$ is the plan that produces the shortest production time for this part. However, $O_b$ is the plan that produces the tightest machining tolerances.

Tables 2 and 3 present the estimated achievable tolerances for operation plan $O_a$ and $O_b$ respectively. Note that because of setup changes between the operations in operation plan $O_a$, the achievable angularity tolerances between $u_1$ and $u_2$, and between $u_1$ and $u_3$, are worse than in $O_b$. If designer had specified a tighter angularity tolerance, then $O_a$ would have not been able to achieve that tolerance.

7.2 Estimating Production Cost and Time
The total time of a machining operation consists of two components, the cutting time (during which the tool is actually engaged in machining), and the non-cutting time (which includes the tool-change time, setup time, etc.). Methods have been developed for estimating the fixed and variable costs of machining operations; conventional formulas for estimating these costs can be found in standard handbooks related to machining economics, such as [32, 31]. The particular formulas we use to evaluate the production cost and time for machining processes are presented in [37, 18].
Table 2
Achievable tolerances for operation plan $O_a$

<table>
<thead>
<tr>
<th>Surface(s)</th>
<th>Tolerance Type</th>
<th>Operations</th>
<th>Design</th>
<th>Achievable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>flatness</td>
<td>$oa_{a1}$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$u_1, u_2$</td>
<td>angularity</td>
<td>$oa_{a1}, oa_{a4}$</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$u_1, u_3$</td>
<td>angularity</td>
<td>$oa_{a1}, oa_{a5}$</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>$u_4, u_5$</td>
<td>length</td>
<td>$oa_{a3}$</td>
<td>$+0.15, -0.15$</td>
<td>$+0.10, -0.10$</td>
</tr>
<tr>
<td>$u_6$</td>
<td>diameter</td>
<td>$oa_{a6}$</td>
<td>$+0.25, -0.00$</td>
<td>$+0.20, -0.00$</td>
</tr>
</tbody>
</table>

Table 3
Achievable tolerances for operation plan $O_b$

<table>
<thead>
<tr>
<th>Surface(s)</th>
<th>Tolerance Type</th>
<th>Operations</th>
<th>Design</th>
<th>Achievable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>flatness</td>
<td>$ob_{b1}$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$u_1, u_2$</td>
<td>angularity</td>
<td>$ob_{b1}, ob_{b4}$</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$u_1, u_3$</td>
<td>angularity</td>
<td>$ob_{b1}, ob_{b5}$</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>$u_4, u_5$</td>
<td>length</td>
<td>$ob_{b3}$</td>
<td>$+0.15, -0.15$</td>
<td>$+0.10, -0.10$</td>
</tr>
<tr>
<td>$u_6$</td>
<td>diameter</td>
<td>$ob_{b6}$</td>
<td>$+0.25, -0.00$</td>
<td>$+0.20, -0.00$</td>
</tr>
</tbody>
</table>

Table 4
Time estimates for plan $O_a$

<table>
<thead>
<tr>
<th>Operation</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$oa_{a1}$</td>
<td>1.42</td>
</tr>
<tr>
<td>$oa_{a2}$</td>
<td>2.08</td>
</tr>
<tr>
<td>$oa_{a3}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$oa_{a4}$</td>
<td>0.50</td>
</tr>
<tr>
<td>$oa_{a5}$</td>
<td>0.42</td>
</tr>
<tr>
<td>4 setup changes</td>
<td>6.0</td>
</tr>
<tr>
<td>6 tool changes</td>
<td>1.0</td>
</tr>
<tr>
<td>Total time</td>
<td>12.09</td>
</tr>
</tbody>
</table>

Table 5
Time estimates for plan $O_b$

<table>
<thead>
<tr>
<th>Operation</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ob_{b1}$</td>
<td>1.42</td>
</tr>
<tr>
<td>$ob_{b2}$</td>
<td>2.08</td>
</tr>
<tr>
<td>$ob_{b3}$</td>
<td>0.25</td>
</tr>
<tr>
<td>$ob_{b4}$</td>
<td>0.50</td>
</tr>
<tr>
<td>$ob_{b5}$</td>
<td>0.42</td>
</tr>
<tr>
<td>2 setup changes</td>
<td>3.0</td>
</tr>
<tr>
<td>4 tool changes</td>
<td>0.66</td>
</tr>
<tr>
<td>Total time</td>
<td>14.17</td>
</tr>
</tbody>
</table>

Examples. Tables 4 and 5 present time estimates for operation plans $O_a$ and $O_b$. In estimating the production time for milling operations, we have added the half the tool diameter to each slot and face length to account for lead-in and break-through. We assume that the part will held in a vise. The setup-change time for the vise is taken from [31]. Although we can similarly estimate the production costs for $O_a$ and $O_b$, we omit this for the sake of brevity.

7.3 Tradeoffs

From the above calculations, it is clear that which of the two operation plans is preferable will depend on the machining tolerances and cost objectives.

Operation plan $O_b$ involves fewer setups than operation plan $O_a$, thus offering an opportunity to achieve a higher machining accuracy. In particular, $O_b$ will be preferable when a tight angularity tolerance is required. But because of the number of passes required for machining $u_2$ and $u_3$, operation plan $O_b$ requires a larger production time than $O_a$. 
When the angularity tolerance requirement is not tight (as is the case for the design specifications shown in Fig. 6), the main objective in process planning may be to achieve a low production time while maintaining an acceptable machining accuracy. Under such circumstances, \( O_a \) will be preferable.

8. **RESEARCH ISSUES**

8.1 **Generating Redundant FBMs**

It is often desirable to use a roughing operation to remove a volume of material followed by a finishing operation in which the swept volume of the tool completely subsumes the removal volume of the roughing operation. Examples are (i) making a hole by drilling and then reaming the hole and (ii) making a slot with a roughing end mill and then finishing the slot with a slightly larger finish end mill.

It follows that redundant FBMs must be considered at some point. The procedure described in this chapter does not allow redundant FBMs at any point. The redundant FBMs should certainly be generated before a cutting order is established and the cost is estimated. (For example, if we are drilling and boring a dozen similar holes in a workpiece, the lowest-cost order is to drill them all then bore them all).

8.2 **Alternative FBMs for Different-Sized Tools**

If we use machining features to represent the swept volume of the cutting portion of the tool, then we will need take into account the possibility of using different tools when we generate alternative FBMs. For example:

1. If we are cutting a pocket whose outline is an hourglass shape (or any shape with a bottleneck in it), the cost-effective method is to use a large tool to cut the bottom and top of the hourglass and a small tool to cut the narrow part in the middle where the large tool would not fit. Using the small tool to cut the entire pocket would take too much time. Thus, a machining-feature decomposition must include three machining features for cutting the pocket.

2. If a large pocket contains tight corners into which a large tool will not fit, a large machining feature should be generated in which the tight corners are rounded, and each tight corner should have its own small machining feature. A small tool should be used for the large machining feature, and small tools for the small machining features.

3. If a machining feature is defined for removing some delta volume, in some cases the corners of the machining feature may have radii assigned to them arbitrarily. A smaller radius lets a smaller machining feature be defined (which helps avoid interferences) but requires a small tool, while a larger radius allows a larger tool to be used. Some heuristic rules are needed to determine radii when generating an FBM.
8.3 Setups

Our current approach does not deal with the machinability considerations involved with setting up the machine tool in order to perform the machining operations. Addressing this issue is a major problem for future work.

9. CONCLUSIONS

In this chapter, we have described a systematic way to generate and evaluate alternative operation plans for a given design. This work represents a step toward the following long-term goals:

1. Providing information about alternative ways in which the part might be machined. We hope this information will aid process engineers or process planning systems in developing alternative process plans, so that the most appropriate plan can be selected depending upon machine tool availability and/or other constraints specific to plant facilities.

2. Pushing process engineering upstream, by providing information about the manufacturability of the design. We hope this information can help designers modify the design if necessary to balance the need for efficient machining against the need for a quality product.

Some of the benefits of our approach are listed below:

1. Since we consider various alternative ways of machining the part, this allows us to consider how well each one balances the need for a quality product against the need for efficient manufacturing.

2. Our approach is based on theoretical foundations which we hope will enable us to make rigorous statements about the soundness, completeness, efficiency, and robustness of the approach.

We anticipate that the results of this work will be useful in providing a way to speed up the evaluation of new product designs in order to decide how or whether to manufacture them. Such a capability will be especially useful in flexible manufacturing systems, which need to respond quickly to changing demands and opportunities in the marketplace.

ACKNOWLEDGEMENT

We wish to thank Tom Kramer for his many helpful comments.

REFERENCES


**APPENDIX**

Table 7 gives details of operation plan $O_a$ for machining Design 1, and Table 8 gives details of operation plan $O_b$ for machining Design 2. Various tools used in these plans are described in Table 6.

<table>
<thead>
<tr>
<th>Tool number</th>
<th>Tool type</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD1</td>
<td>HSS STD drill</td>
<td>tool dia = 10 mm</td>
</tr>
<tr>
<td>EM1</td>
<td>HSS end mill</td>
<td>tool dia = 40 mm, number of teeth = 4</td>
</tr>
<tr>
<td>EM2</td>
<td>HSS end mill</td>
<td>tool dia = 10 mm, number of teeth = 4</td>
</tr>
<tr>
<td>EM3</td>
<td>HSS end mill</td>
<td>tool dia = 20 mm, number of teeth = 4</td>
</tr>
<tr>
<td>FM1</td>
<td>HSS face mill</td>
<td>tool dia = 40 mm, number of teeth = 4</td>
</tr>
</tbody>
</table>
### Table 7
Details of operation plan $O_a$

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Tool</th>
<th>Parameters</th>
<th>Feed and speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$oa_h$</td>
<td>hole drilling</td>
<td>TD1</td>
<td>hole dia = 10 mm, hole length = 85 mm</td>
<td>feed = 0.10 mm/rev RPM = 600</td>
</tr>
<tr>
<td>$oa_{s1}$</td>
<td>end milling</td>
<td>EM1</td>
<td>slot width = 30 mm, slot length = 80 mm</td>
<td>feed = 0.20 mm/tooth RPM = 300</td>
</tr>
<tr>
<td>$oa_{s2}$</td>
<td>end milling</td>
<td>EM2</td>
<td>slot width = 10 mm, slot length = 55 mm</td>
<td>feed = 0.10 mm/tooth RPM = 1200</td>
</tr>
<tr>
<td>$oa_{s3}$</td>
<td>end milling</td>
<td>EM3</td>
<td>slot width = 20 mm, slot length = 50 mm</td>
<td>feed = 0.15 mm/tooth RPM = 600</td>
</tr>
<tr>
<td>$oa_{s4}$</td>
<td>face milling</td>
<td>FM1</td>
<td>face width = 30 mm, face length = 55 mm</td>
<td>feed = 0.30 mm/tooth RPM = 600</td>
</tr>
<tr>
<td>$oa_{s5}$</td>
<td>face milling</td>
<td>FM1</td>
<td>face width = 30 mm, face length = 55 mm</td>
<td>feed = 0.30 mm/tooth RPM = 600</td>
</tr>
</tbody>
</table>

### Table 8
Details of operation plan $O_b$

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Tool</th>
<th>Parameters</th>
<th>Feed and speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ob_h$</td>
<td>hole drilling</td>
<td>TD1</td>
<td>hole dia = 10 mm, hole length = 85 mm</td>
<td>feed = 0.10 mm/rev RPM = 600</td>
</tr>
<tr>
<td>$ob_{s1}$</td>
<td>end milling</td>
<td>EM1</td>
<td>slot width = 30 mm, slot length = 80 mm</td>
<td>feed = 0.20 mm/tooth RPM = 300</td>
</tr>
<tr>
<td>$ob_{s2}$</td>
<td>end milling</td>
<td>EM2</td>
<td>slot width = 10 mm, slot length = 55 mm</td>
<td>feed = 0.10 mm/tooth RPM = 1200</td>
</tr>
<tr>
<td>$ob_{s3}$</td>
<td>end milling</td>
<td>EM3</td>
<td>slot width = 20 mm, slot length = 50 mm</td>
<td>feed = 0.15 mm/tooth RPM = 600</td>
</tr>
<tr>
<td>$ob_{s4}$</td>
<td>end milling</td>
<td>EM1</td>
<td>slot width = 15 mm, slot length = 30 mm</td>
<td>feed = 0.20 mm/tooth RPM = 300</td>
</tr>
<tr>
<td>$ob_{s5}$</td>
<td>end milling</td>
<td>EM1</td>
<td>slot width = 15 mm, slot length = 30 mm</td>
<td>feed = 0.20 mm/tooth RPM = 300</td>
</tr>
</tbody>
</table>