Supplemental Exercises: Unit 6
Scientific Computing with Case Studies
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1. Consider solving a nonlinear system of equations using the limited memory quasi-Newton method using Broyden's update formula with $\boldsymbol{B}^{(0)} = \boldsymbol{I}$:

$$\boldsymbol{B}^{(k+1)} = \boldsymbol{B}^{(k)} + \frac{(\boldsymbol{y}^{(k)} - \boldsymbol{B}^{(k)}\boldsymbol{s}^{(k)})\boldsymbol{s}^{(k)T}}{\boldsymbol{s}^{(k)T}\boldsymbol{s}^{(k)}}.$$

As an example, let $k = 2$.

(a) Let

$$(\boldsymbol{B}^{(k+1)})^{-1} = (\boldsymbol{B}^{(k)})^{-1} + \boldsymbol{w}^{(k)}\boldsymbol{u}^{(k)T}.$$

Use the Sherman-Morrison-Woodbury formula

$$(\boldsymbol{A} - \boldsymbol{Z}\boldsymbol{V}^T)^{-1} = \boldsymbol{A}^{-1} + \boldsymbol{A}^{-1}\boldsymbol{Z}(\boldsymbol{I} - \boldsymbol{V}^T\boldsymbol{A}^{-1}\boldsymbol{Z})^{-1}\boldsymbol{V}^T\boldsymbol{A}^{-1}$$

to write formulas for the vectors $\boldsymbol{w}^{(k)}$ and $\boldsymbol{u}^{(k)}$.

(b) What vectors would you store in order to be able to form $(\boldsymbol{B}^{(3)})^{-1}\boldsymbol{v}$ for an arbitrary vector $\boldsymbol{v}$?

(c) How many floating-point multiplications would it take to form $(\boldsymbol{B}^{(3)})^{-1}\boldsymbol{v}$?
