Supplemental Exercises: Unit 7
Scientific Computing with Case Studies
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1. Consider the matrix $\boldsymbol{A}$ with sparsity pattern

$$\begin{bmatrix} \times & 0 & \times & \times & 0 & 0 \\ 0 & \times & 0 & \times & 0 & \times \\ \times & 0 & \times & 0 & 0 & 0 \\ \times & \times & 0 & \times & 0 & \times \\ 0 & 0 & 0 & 0 & \times & 0 \\ 0 & \times & 0 & \times & 0 & \times \end{bmatrix}.$$

(a) Draw the graph corresponding to the matrix.

(b) Reorder the matrix using the Cuthill-McKee algorithm. Count the number of nonzeros in the resulting Cholesky factors and compare with the original ordering.

(c) Reorder the matrix using the minimum degree algorithm. Count the number of nonzeros in the resulting Cholesky factors and compare with the original ordering.

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2. (a) Create a linear system of equations $\boldsymbol{Ax} = \boldsymbol{b}$ where $\boldsymbol{A}$ is 2×2. Graph the two equations.

(b) Using $\boldsymbol{x} = [1, 1]^T$ as a starting guess, illustrate on the graph the result of running two steps of the Gauss-Seidel algorithm.

(c) Is the algorithm convergent for your problem? Justify your answer by determining $\boldsymbol{G}$ and its eigenvalues.

(d) Repeat parts (a) through (c) with a different linear system: one for which Gauss-Seidel converges if your first one did not, and one for which Gauss-Seidel diverges if your first one converges.
