AMSC 607 / CMSC 764 Homework 1, Fall 2008
30 points
Due September 16, 2pm

1. (3) At what rate does the sequence

$$\boldsymbol{e}_k = 1 + (0.5)^{2^k}$$

converge to 1?

2. (5) Let $\boldsymbol{x} = \boldsymbol{S}\boldsymbol{z} + \boldsymbol{d}$, where $\boldsymbol{S}$ is a given $n \times n$ matrix and $\boldsymbol{d}$ is a given $n \times 1$ vector.

Suppose f is a function of n variables, and define $\hat{f}(\boldsymbol{z}) = f(\boldsymbol{x}) = f(\boldsymbol{S}\boldsymbol{z} + \boldsymbol{d})$. Write expressions for the gradient and Hessian of $\hat{f}$ with respect to the variables $\boldsymbol{z}$, using the gradient and Hessian of the function f. (Hint: compute $\partial \hat{f} / \partial z_j$ by using the chain rule and the values $\partial f / \partial x_i$ and $\partial x_i / \partial z_j$.)

3. Consider the following problem: Find a value of γ so that the solution $\boldsymbol{p}$ to the linear system

$$(\boldsymbol{H} + \gamma \boldsymbol{I})\boldsymbol{p} = -\boldsymbol{g}$$

satisfies $\|\boldsymbol{p}\|_2 = \delta$, where $\delta > 0$ is a given value.

Suppose we have factored $\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T$, where $\boldsymbol{\Lambda}$ is a diagonal matrix with diagonal elements λ_i and $\boldsymbol{U}$ is orthogonal, so that $\boldsymbol{U}\boldsymbol{U}^T = \boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$.

3a. (5) Write the solution $\boldsymbol{p}$ to the linear system in terms of $\boldsymbol{g}$, γ, $\boldsymbol{U}$, $\boldsymbol{u}_i$, and λ_i, where $\boldsymbol{u}_i$ is the ith column of $\boldsymbol{U}$. (Hint: Remember that scalars like γ commute with matrices.)

3b. (2) Show that, for any vector $\boldsymbol{w}$, $\|\boldsymbol{w}\|_2 = \|\boldsymbol{U}^T\boldsymbol{w}\|_2$.

3b. (5) Use 3b to write an expression for $\|\boldsymbol{p}\|_2$ in terms of $\boldsymbol{g}$, γ, $\boldsymbol{U}$, and λ_i.

3c. (5) Find an interval for γ on which $\|\boldsymbol{p}\|_2$ is monotonically decreasing. (Hint: Remember that some of the λ_i might be negative.)

3d. (5) Describe how you could use MATLAB's `fzero` to find a value of γ for which $\|\boldsymbol{p}\|_2 = \delta$ (if such a value exists). What initial interval would you give `zeroin`?

In Homework 2, you will write a program, using this algorithm to solve minimization problems.