AMSC 607 / CMSC 764 Homework 8, Fall 2010
Due November 9, before class begins.

11.

11a. (4) Apply affine scaling to the primal linear programming problem

$$\min_{\boldsymbol{x}} \boldsymbol{c}^T \boldsymbol{x}$$

$$\begin{aligned} \boldsymbol{A}\boldsymbol{x} &= \boldsymbol{b}, \\ \boldsymbol{x} &\geq \boldsymbol{0}, \end{aligned}$$

but instead of mapping $\boldsymbol{x}^{(k)}$ to $\boldsymbol{e}$ (as we did in class), map it to $\boldsymbol{D}^{1/2}\boldsymbol{e}$, where $\boldsymbol{D}$ is a positive definite diagonal matrix. Display $\bar{\boldsymbol{A}}$, $\bar{\boldsymbol{c}}$, $\boldsymbol{\Delta}\bar{\boldsymbol{x}}$ and the resulting $\boldsymbol{\Delta x}$.

11b. (8) Apply affine scaling to the dual linear programming problem

$$\max_{\boldsymbol{y}} \boldsymbol{b}^T \boldsymbol{y}$$

$$\begin{aligned} \boldsymbol{A}^T\boldsymbol{y} + \boldsymbol{z} &= \boldsymbol{c} \\ \boldsymbol{z} &\geq \boldsymbol{0} \end{aligned}$$

mapping $\boldsymbol{z}^{(k)}$ to $\boldsymbol{De}$. (The vector $\boldsymbol{y}$ is left unchanged, since it is not required to be nonnegative.) Note that the constraint matrix for the dual problem is $[\boldsymbol{A}^T, \boldsymbol{I}]$, and remember to include the term $\boldsymbol{0}^T\boldsymbol{z}$ in the objective function. Display the resulting transformed constraint matrix, transformed gradient of the objective function, and the resulting $\boldsymbol{\Delta y}$ and $\boldsymbol{\Delta z}$.

11c. (8) Consider the quadratic programming problem

$$\min_{\boldsymbol{x}} \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} - \boldsymbol{x}^T\boldsymbol{c}$$

$$\begin{aligned} \boldsymbol{A}\boldsymbol{x} &= \boldsymbol{b}, \\ \boldsymbol{x} &\geq \boldsymbol{0}, \end{aligned}$$

where $\boldsymbol{Q}$ is symmetric and positive definite. Write the optimality conditions for the problem, and write the normal equations corresponding to applying one step of Newton's method to the optimality conditions. (The normal equations for linear programming are on p.8 of the notes.)
