

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, communication with others, scratchpaper, etc.

Name \_\_\_\_\_

1. (10) Compute the quadratic polynomial that interpolates the data

$$(x, f(x)) = (0, 5), (1, 11), (2, 21).$$

Use either the Lagrange form or the Newton form.

2. (10) Suppose we are interested in approximating a function  $f(x)$  on the interval  $[-1, 1]$  using a polynomial  $p_{n-1}$  that interpolates  $f$  at the  $n$  points given by  $-1, -1+h, -1+2h, \dots, 1$ , where  $h = 2/(n-1)$ . Suppose you know that, on this interval, the maximum absolute value of all derivatives of  $f$  is 25:

$$\max_{x \in [-1, 1]} |f^{(k)}(x)| < 25, \quad k = 0, 1, \dots$$

Describe how you would determine how many interpolation points you should use to guarantee that

$$|f(x) - p_{n-1}(x)| \leq 10^{-3}, \quad \text{for all } x \in [-1, 1].$$

In particular, **write a sequence of Matlab statements** to verify that a particular value of  $n$  was large enough.

A useful formula:

$$f(x) - p_{n-1}(x) = \frac{f^{(n)}(\xi)}{n!} (x - x_1) \dots (x - x_n).$$