AMSC/CMSC 498 Class Participation Exercise 3a , Spring 2012

Name ______________________________________

(Possible score: 4 points.)

1. Let $\boldsymbol{U\Sigma V}^T$ be the singular value decomposition of the $n \times n$ matrix $\boldsymbol{A}$. Tell me as much as you can about $\boldsymbol{U}$, $\boldsymbol{\Sigma}$, and $\boldsymbol{V}$.

Answer:

- The three matrices all have dimensions $n \times n$.
- $\boldsymbol{\Sigma}$ is diagonal, and the entries σ_i along the diagonal are nonnegative with $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n$.
- The 2-norm of each column of $\boldsymbol{U}$ and $\boldsymbol{V}$ is one.
- Let $\boldsymbol{u}_i$ be the ith column of $\boldsymbol{U}$. Then $\boldsymbol{u}_i^T \boldsymbol{u}_j = 0$ if $i \neq j$. The same property holds for the columns of $\boldsymbol{V}$.
- $\boldsymbol{U}\boldsymbol{U}^T = \boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$, so $\boldsymbol{U}^{-1} = \boldsymbol{U}^T$.
- $\boldsymbol{V}\boldsymbol{V}^T = \boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{I}$, so $\boldsymbol{V}^{-1} = \boldsymbol{V}^T$.

2. Complete the following formula:

$$\boldsymbol{A} = \sum_{i=1}^{n} \sigma_i \ldots.$$

Answer:

$$\boldsymbol{A} = \sum_{i=1}^{n} \sigma_i \boldsymbol{u}_i \boldsymbol{v}_i^T.$$