TABLE II
OPTIMAL DISPATCHING FOR $p = 0.1$ AND $p = 0.25$

	p = .1 (α = 2.5)	p = .25 (α = 6.25)
ATOM	FACILITY TO BE DISPATCHED	FACILITY TO BE DISPATCHED
1	a	a
2	a	a
3	a	c
4	b	b
5	b	b
6	a	a
7	c	c
8	d	d
9	50% of the times b;50% of the times e †	e
10	c	c
11	b	b
12	d	d
13	c	c
14	b or d	b or d
15	d	25% of the times b ; 75% of the times d
16	b	b
17	c (and b)†	b
18	d (and e)†	e
19	c	c
20	c	c
21	c or d*	c or d*
22	b	b
23	d	d
24	e	e
25	b or d*	b or d*

*Choice can be done arbitrarily.
†Other optimal solutions exist where either 17 or 18 obtains mixed service.

facility b). Table II includes also the optimal solution for $p = 25$ percent.

V. SUMMARY

We have considered problems with two objectives, which are quite typical to several service systems. The first objective took into account the cost of operating the system whereas the second one was a measure of "customers' suffer" that is required to be less than some unbearable level T.

Two closely related models were included. In one problem we minimized the expected cost of operating the system subject to an upper bound constraint on the percentage of customers that will not be reached within time period T while in the second model we did the opposite. An exact and fast algorithm (linear in the dimensions of the problem) was presented for the first model. For medium size problems the algorithms can even be solved manually. The same algorithm can be used to solve the integer version of the problem.

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An Optimization Approach to Edge Reinforcement

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***Abstract*—A steepest descent method is used to iteratively adjust edge magnitudes and thereby enhance the distinction between edge and nonedge pixels. The results appear to be better than those obtained from relaxation methods based on edge probabilities, using either Bayesian probability adjustment or optimization methods.**

I. INTRODUCTION

During the past five years, a number of iterative methods of edge reinforcement have been proposed. VanderBrug [1] used iterative reinforcement of the magnitude responses obtained by a local edge detection operation, based on the response magnitudes (if any) at a set of neighbors depending on the edge direction; he also iteratively adjusted the direction based on the directions of the responses at these neighbors. Eberlein [2] used an iterative competition process to thin edge responses, after first blurring them to reduce gaps.

During the same period, the probabilistic relaxation approach to pixel classification was developed and applied to various image segmentation tasks, including edge detection. In this approach, edge magnitudes are interpreted as probabilities, and are iteratively adjusted based on the probabilities at the neighbors. For early examples of this approach see Schachter *et al.* [3] and Hanson and Riseman [4]. The iterative probability adjustment process is usually designed on the basis of conditional probability arguments (e.g., Peleg [5]). An alternative basis for probability adjustment is an optimization approach, investigated by Faugeras and Berthod [6].

This correspondence investigates an optimization approach to edge reinforcement that does not involve probabilistic concepts. A cost function is defined, based on the local pattern of edge responses, which is low for sharp, strong edges, and a steepest descent method is used to adjust the edge magnitudes so as to reduce the value of this cost function.[1] The edge reinforcement results obtained in this way seem to be better than those obtained by relaxation methods, whether based on conditional probability or on optimization. For an analogous optimization approach to a different segmentation task, namely thresholding, see Narayanan *et al.* [8].

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[1]Such a cost function can also be used to evaluate edge detector responses; see Kitchen and Rosenfeld [7].

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II. Method

A. Initial Values

The initial edge magnitude and direction at each pixel are obtained using the Sobel operator. In the neighborhood

$$\begin{matrix} A & B & C \\ D & E & F \\ G & H & I \end{matrix}$$

of pixel E, we define

$$\Delta_x \equiv \tfrac{1}{4}[(A+2D+G)-(C+2F+I)]$$
$$\Delta_y \equiv \tfrac{1}{4}[(A+2B+C)-(G+2H+I)].$$

The Sobel gradient magnitude and direction are then

$$\sqrt{\Delta_x^2+\Delta_y^2}$$

and

$$\tan^{-1}(\Delta_y/\Delta_x),$$

respectively. The edge direction is at right angles to the gradient direction. For brevity, we let m_i and α_i denote the magnitude and direction of the edge response at the ith pixel.

B. Cost Function

The cost function is a linear combination of two components

$$C=(1-\beta)C_1+\beta C_2.$$

Here component C_1 measures the inconsistency between the edge responses at the given pixel and its neighbors, while component C_2 measures the ambiguity of the responses, giving low cost to responses that are either high or low. The detailed definitions of C_1 and C_2 are given in the following paragraphs.

Let

$$\bar{m}_i=\sum_{j\in N_i} m_j\gamma_{ij}\Big/\sum_{j\in N_i}\gamma_{ij},$$

where N_i is the neighborhood of the ith pixel, and

$$\gamma_{ij}\equiv\left[1-\frac{|\alpha_i-\alpha_j|}{\pi}\right]\left[1-\frac{|\alpha_i-\partial_{ij}|}{\pi/2}\right]^2,$$

Here ∂_{ij} is the direction from the ith pixel to the jth pixel. The first factor in γ_{ij} measures the similarity in the edge directions at the ith and jth pixels, and is 1 when these directions are the same. The second factor similarly measures the similarity between the direction to the jth pixel and the edge direction at the ith pixel.[2] Thus $\gamma_{ij}=1$ if and only if the edges are collinear. Hence $\bar{m}_i$ is a linear combination of the edge magnitudes in the neighborhood of the ith pixel, weighted by their collinearities with the edge direction at the ith pixel. We now define

$$C_1\equiv\sum_i(m_i-\bar{m}_i)^2.$$

The ith term of C_1 measures the inconsistency between the edge magnitude at the ith pixel and the edge magnitudes in its neighborhood. In our experiments, a 3×3 neighborhood was used.

Let t be an edge detection threshold (i.e., we say that the ith pixel is an edge pixel if and only if $m_i \geqslant t$). Unambiguous edge responses should be either much higher or much lower than t, not close to t. We now define

$$C_2\equiv-\sum_i(m_i-t)^2.$$

[2]Squaring the factor gives it greater weight, which was found empirically to give better results.

The ith term of C_2 measures the ambiguity of the edge magnitude m_i (relative to the threshold t); it is highest ($=0$) when $m_i=t$, and is low for m_i either much larger or much smaller than t.

C. Optimization Procedure

We use a steepest descent procedure to minimize C:

$$\frac{\partial C_1}{\partial m_i}=2(m_i-\bar{m}_i)-\sum_{j\in N_i}2(m_j-\bar{m}_j)\left[\frac{\partial}{\partial m_i}\bar{m}_j\right]$$
$$=2(m_i-\bar{m}_i)-\sum_{j\in N_i}2(m_j-\bar{m}_j)\frac{\gamma_{ji}}{\sum_{k\in N_i}\gamma_{jk}}$$
$$\frac{\partial C_2}{\partial m_i}=-2(m_i-t)$$
$$m_i^{(k+1)}=m_i^{(k)}-\lambda^{(k)}\frac{\partial C}{\partial m_i},$$

where $\lambda^{(k)}$ is chosen so as to minimize C along the gradient direction and such that the values of m_i lie within range, and where N_i is the neighborhood of i.

III. Results

Fig. 1 shows the input image (a portion of a Landsat scene) used in most of our experiments, together with the initial edge magnitudes. We used parameter values t in the range 12–16 and β in the range 0.5–0.9 in defining the cost function C. Higher values of t were found to eliminate too many edges, causing gaps in the output, while lower values allow too many low-magnitude edges and tend to thicken the reinforced edges. Higher values of β make the results look too much like the thresholded output, while lower values do more noise cleaning and increase the convergence time.

Fig. 2 shows the results of 10 iterations of the steepest descent procedure for $t=12$, 14, and 16, using $\beta=0.5$ (top row), 0.7 (middle row), and 0.9 (bottom row). Fig. 3 shows the results of iterations 1, 2, 3, 5, 7, and 10 for $\beta=0.5$ and $t=12$.

Fig. 4 shows results using the relaxation process of [3], with coefficients $C1=0.866$, $C2=C3=0.005$, $C4=0.124$, $W=3$ (see [3]), after iterations 1, 2, 3, 5, 7, and 10. Note that the edges become thicker. The results are sensitive to the choice of the coefficients (C's). Fig. 5 shows results using the optimization relaxation process of [6], with $\alpha=0.2$ and 0.8 (see [6]) after 10 iterations; the top row shows initial probabilities. The maximum edge probability is displayed (as pixel brightness) if it is greater than the no-edge probability; otherwise, the pixel is left black. For obtaining the initial probabilities, a global threshold of 30 was used. An alternative (see [6]) is to use a local threshold, equal to the (3×3) neighborhood average, for each pixel; this yields the results shown in Fig. 6 ($\alpha=0.2$ and 0.8, 10 iterations, initial probabilities at top), which are quite poor.

A second example, involving a different type of image (a portion of a white blood cell), is shown in [9]; the results are entirely analogous.

The question of how many times the optimization procedure should be iterated is not addressed here, but as Fig. 3 shows, there is little change after the first few iterations. One possibility is to measure the rate of change at each iteration, and stop when it drops below some fraction of the initial rate of change; see [10].

The computational cost of the optimization scheme used here is relatively high, as compared to that of probabilistic relaxation; but in many situations this cost would be justified by the superior performance of the present method.

IV. Concluding Remarks

The results obtained by our method are visually at least as good as those obtained using either of the two relaxation ap-

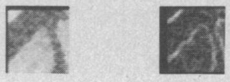

Fig. 1

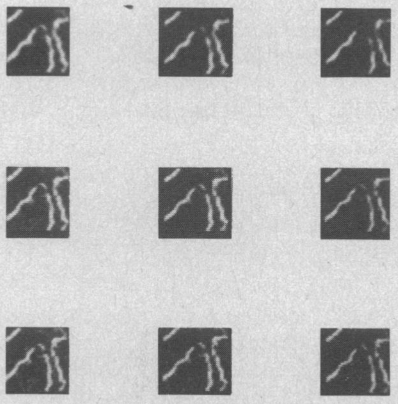

Fig. 2

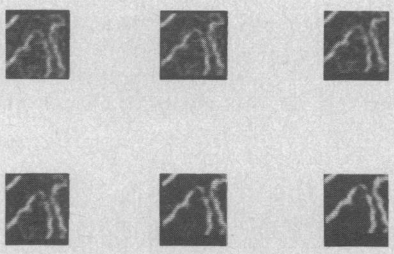

Fig. 3

Fig. 4

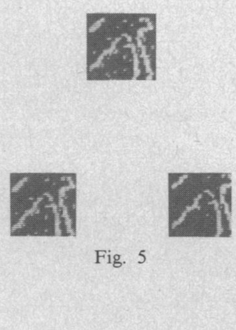

Fig. 5

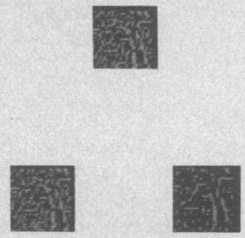

Fig. 6

proaches; this can be seen by comparing the results shown in the figures. Moreover, our method does not involve the introduction of probabilistic concepts. Further improvement could probably be obtained by adjusting the edge direction as well as the edge magnitude, but this was not done in order to avoid additional computational cost.

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Segmentation of FLIR Images: A Comparative Study

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***Abstract*—Several segmentation techniques were applied to a set of 51 FLIR (Forward-Looking InfraRed) images of four different types, and the results were compared to hand segmentations. There were substantial differences in performance, indicating that the choice of proper technique is very important. The segmentation techniques used were "superslice," "pyramid spot detection," two versions of "relaxation," "pyramid linking,"**

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