

Proof sketch that Manson/Pugh allows reordering

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Consider a program P and the program P' that is obtained from P by reordering two adjacent statements x and y . Let x be the statement that comes before y in P , and after y in P' . The statements x and y may be any two statements such that

- reordering x and y doesn't eliminate any happens-before edges, except for reversing the one between x and y ,
- x and y are not conflicting accesses to the same variable,
- x and y are not both synchronization actions, and
- the intra-thread semantics of x and y allow reordering (e.g., x doesn't store into a register that is read by y).

Assume that we have a valid execution E' of program P' . To show that the transformation of P into P' is legal, we need to show that there is a valid execution E of P that has the same observable behavior as E' .

Assume $E' = \langle S, so, hb', co \rangle$. We are going to show that $E = \langle S, so, hb, co \rangle$ is also a valid execution of P . Let a_x and a_y denote the actions corresponding to the statements x and y . Because of the reordering the happens-before ordering may be different but we know that $hb - \{a_x \rightarrow a_y\} \subseteq hb' - \{a_y \rightarrow a_x\}$. Clearly, if E' is consistent then E is consistent, so we only need to worry about showing that co is justified as the causal order of E .

- Assume that $co = \alpha a_y \beta a_x \gamma$.

We don't need to worry about any actions that were prescient in E' . The justification of those prescient actions in E' will also justify them in E .

The only action that could be prescient in E but not E' is a_y . If a_y is not prescient in E' , we know a_x is the only action that comes after a_y in the causal order such that $a_x \xrightarrow{hb} a_y$, and that if a_y is a read, it sees a value in α . In order to justify the prescient action a_y in E , we need to show that for each execution of P whose causal order starts with α , the action a_y is allowed to occur and if a_y is a read that a_y can see a write of the value seen by a_y in E .

We know intra-thread semantics will cause a_y to occur, since all actions other than a_x that occur before a_y in program order are in α , and we have assumed as a condition for the reordering that a_x does not effect the intra-thread semantics of a_y . If a_y is a read that sees a write w in α , we know it can see w in any execution with a causal order that starts with α .

- Alternatively, assume that $co = \alpha a_x \beta a_y \gamma$. Then any action in E that is prescient is also prescient in E' , and the justifications used to show that those actions are justified in E' will also show that those actions are justified in E .