Proof sketch that Manson/Pugh allows reordering

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Consider a program $P$ and the program $P'$ that is obtained from $P$ by reordering two adjacent statements $x$ and $y$. Let $x$ be the statement that comes before $y$ in $P$, and after $y$ in $P'$. The statements $x$ and $y$ may be any two statements such that

- reordering $x$ and $y$ doesn’t eliminate any happens-before edges, except for reversing the one between $x$ and $y$,
- $x$ and $y$ are not conflicting accesses to the same variable,
- $x$ and $y$ are not both synchronization actions, and
- the intra-thread semantics of $x$ and $y$ allow reordering (e.g., $x$ doesn’t store into a register that is read by $y$).

Assume that we have a valid execution $E'$ of program $P'$. To show that the transformation of $P$ into $P'$ is legal, we need to show that there is a valid execution $E$ of $P$ that has the same observable behavior as $E'$.

Assume $E' = (S, so, hb', co)$. We are going to show that $E = (S, so, hb, co)$ is also a valid execution of $P$. Let $a_x$ and $a_y$ denote the actions corresponding to the statements $x$ and $y$. Because of the reordering the happens-before ordering may be different but we know that $hb - \{a_x \rightarrow a_y\} \subseteq hb' - \{a_y \rightarrow a_x\}$. Clearly, if $E'$ is consistent then $E$ is consistent, so we only need to worry about showing that $co$ is justified as the causal order of $E$.

- Assume that $co = \alpha a_y \beta a_x \gamma$.

  We don’t need to worry about any actions that were prescient in $E'$. The justification of those prescient actions in $E'$ will also justify them in $E$.

  The only action that could be prescient in $E$ but not $E'$ is $a_y$. If $a_y$ is not prescient in $E'$, we know $a_x$ is the only action that comes after $a_y$ in the causal order such that $a_x \xRightarrow{hb} a_y$, and that if $a_y$ is a read, it sees a value in $\alpha$. In order to justify the prescient action $a_y$ in $E$, we need to show that for each execution of $P$ whose causal order starts with $\alpha$, the action $a_y$ is allowed to occur and if $a_y$ is a read that $a_y$ can see a write of the value seen by $a_y$ in $E$.

  We know intra-thread semantics will cause $a_y$ to occur, since all actions other than $a_x$ that occur before $a_y$ in program order are in $\alpha$, and we have assumed as a condition for the reordering that $a_x$ does not effect the intra-thread semantics of $a_y$. If $a_y$ is a read that sees a write $w$ in $\alpha$, we know it can see $w$ in any execution with a causal order that starts with $\alpha$.

- Alternatively, assume that $co = \alpha a_x \beta a_y \gamma$. Then any action in $E$ that is prescient is also prescient in $E'$, and the justifications used to show that those actions are justified in $E'$ will also show that those actions are justified in $E$.  