CMSC427 Fall 2017
Hw 4 – Transformations and vectors SOLUTIONS

1. Using the barn from the PowerPoint presentation on polygonal meshes, sketch what the data structure must look like following the tetrahedron example from the same presentation. Eg, you need to explain the components: vertices, edges, faces. List all the vertices, and all the normals (assume the roof is at 45 degrees), but to keep this from being too tedious, you only need to do two faces: the face with five sides in the plane and normal n6, and the roof face with four sides and normal n2. For the vertices assume the barn is 1 unit wide, 1 unit tall to the roof, and 2 units long. The funky vertices are those on the barn's roof ridge, the funky normals are those for the roof faces.

Vertex list: 10

v0: (0,0,0)	v4: (0,1,0)	v8: (0.5, 1.4, 2)
v1: (1,0,0)	v5: (0,0,2)	v9: (0, 1, 2)
v2: (1,1,0)	v6: (1,0,2)	
v3: (0.5,1.5,0)	v7: (1,1,2)	

Normals: 7

n0: <-1,0,0>	n4: <0,-1,0>
n1: <-0.717,0.717,0>	n5: <0,0,1>
n2: <+0.717,0.717,0>	n6: <0,0,-1>
n3: <+1,0,0>	

Faces: 7 Listed here as (vertex, normal) pairs with índices into above lists

face0: [(0,6), (4,6), (3,6), (2,6), (1,6)]	CCW as viewed down n6, n6 normal for all
face1: [(2,2), (7,2), (8,2), (3,2)]	CCW as viewed down n2, n2 normal for all

2. Compute the camera matrix given these input values. You can check with printCamera, BTW. Show your work by giving the calculation at each step, and then the final matrix.
eye=e=at = (1,1,1) lookAt = (0,0,0) up = <0,1,0>

Solution: (calculations done in Octave-Online)
n = eye-lookAt = <1,1,1>
zc = n = n/norm(n) = <0.5774, 0.5774, 0.5774>
xc = up x zc = <0.3333, -0.6667, 0.3333>
yc = zc x uc
d = < -eye•xc -eye•yc, -eye•zc > = <0,0, -1.7321> (we don't normalize this one)

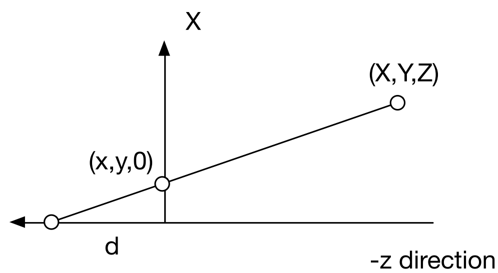
$$M = \begin{bmatrix} xc.x & xc.y & xc.z & d.x \\ yc.x & yc.y & yc.z & d.y \\ zc.x & zc.y & zc.z & d.z \\ 0 & 0 & 1 & d \end{bmatrix} = \begin{bmatrix} 0.70711 & 0 & -0.70711 & 0 \\ -0.40825 & 0.81650 & -0.40825 & 0 \\ 0.57735 & 0.57735 & 0.57735 & -1.7321 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. If an object like Hill's barn is centered at the origin, and like the barn is flat on the (x,z) plane, then give the values of at, LookAt and up that would give you a front view (down the x axis), side view (down the z), and top view (down the y).

Solution: The question gives you some freedom to pick the exact distance from the origin, and the nature of the up vector. For the y axis you can pick for up the z axis (as below) or the x axis.

Down the x axis: at = (100,0,0) LookAt=(0,0,0), up=<0,1,0>
Down the z axis: at = (0,0,100) LookAt = (0,0,0), up=<0,1,0>
Down the y axis: at = (0,100,0) LookAt = (0,0,0), up=<0,0,1> (up can't be y vector)

4. Using similar triangle arguments, give the perspective equations for x and y when the image plane is at z=0, the camera position (the focal point) is at d, the camera is looking out the negative z axis, and the point is at (X,Y,Z). Once you have the equations, then put them into homogenous matrix that gives (x,y,0) = M * (X,Y,Z).



Solution: Setting up similar triangles, we have $\frac{x}{d} = \frac{X}{d-Z}$ so $x = \frac{dX}{d-Z}$

Putting this in matrix form, we want to multiply each coordinate by d, and then have the last row produce d-z.

$$M = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & -1 & d \end{bmatrix}$$

$$M * P = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & -1 & d \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} dX \\ dY \\ dZ \\ d-Z \end{bmatrix}$$

5. Give 2D homogeneous matrices to scale a 2D point P by s=3, to translate a point to the position (3,-4), and then show how you'd use those two matrices to scale an object centered at P about its center. Set up the sequence of matrices as an equation (eg, like P' = M1*M2*P), and then just this once, show the single matrix that results from multiplying these out.

Solution: The two matrices are given by:

$$Ms = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Mt = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

To scale around (3,-4) first you'd translate that point to the origin, scale, then translate back. The matrices are multiplied in the reverse order of that sequence so:

$$M = Mt * Ms * Mt^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = Mt * Ms * Mt^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & -9 \\ 0 & 3 & 12 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -6 \\ 0 & 3 & 8 \\ 0 & 0 & 1 \end{bmatrix}$$

6. What is distance of the point (7,8) from the line defined by P0=(0,0) and v=<2,3>? Show the formula you use.

Solution: Distance from a point to line is on the DotProductExercise handout. The formula given there is

$$\boldsymbol{d} = \left|(\boldsymbol{P} - \boldsymbol{P0}) - \frac{(\boldsymbol{P} - \boldsymbol{P0}) \bullet \boldsymbol{v}}{\boldsymbol{v} \bullet \boldsymbol{v}}\boldsymbol{v}\right|$$

$$\boldsymbol{d} = \left|((\mathbf{7},\mathbf{8}) - (\mathbf{0},\mathbf{0})) - \frac{\big((\mathbf{7},\mathbf{8}) - (\mathbf{0},\mathbf{0})\big) \bullet < \mathbf{2},\mathbf{3} >}{< \mathbf{2},\mathbf{3} > \bullet < \mathbf{2},\mathbf{3} >} < \mathbf{2},\mathbf{3} >\right|$$

$$\boldsymbol{d} = \left|< \mathbf{7},\mathbf{8} > - \frac{< \mathbf{7},\mathbf{8} > \bullet < \mathbf{2},\mathbf{3} >}{\mathbf{13}} < \mathbf{2},\mathbf{3} >\right| = \left|< \frac{\mathbf{15}}{\mathbf{13}}, \frac{\mathbf{10}}{\mathbf{13}} >\right| = \mathbf{1.38675}$$

7. In class we constructed the parametric form for a cylinder with P(u,v) = < rcos u, hv, rsin u>. That the basic form for a surface of revolution. If we want to revolve a parabola around the y axis, given its parametric form in (x,y) as we had earlier in class, what would be the parametric form for the parametric surface of revolution? Assume the parabola is simply $y = x^2$.

Solution: the answer is to make width and height a function of v using a parametric equation for a parabola. Set the parametric equation to x=v, y=v^2, so the width varies with v, and the height by v^2, with r and h retained as scaling factors. Now we have the following with u in [0,2π] and v in [0,1].

P(u,v) = < r v cos u, hv^2, r v sin u>.