CMCS427 Supplementary notes on shading

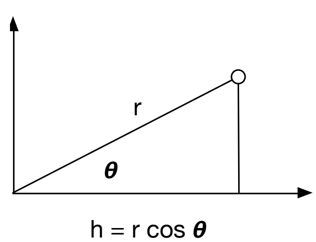
The class PowerPoint L09P1Shading reviews a number of specific computations for lighting and shading, including:

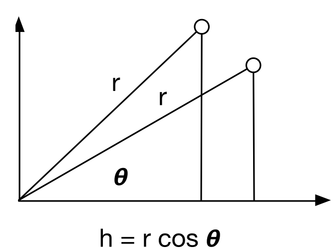
1. Diffuse shading
2a. Specular shading, Phong model
2b. Specular shading, Blinn model (alternative to Phong)
3. Ambient shading
4. Directional light (at infinity)
5. Point source light
6. Spot light

These are combined into the general shading equation, in either Phong or Blinn flavor depending on which is used for specular reflection. Here's details on diffuse shading.

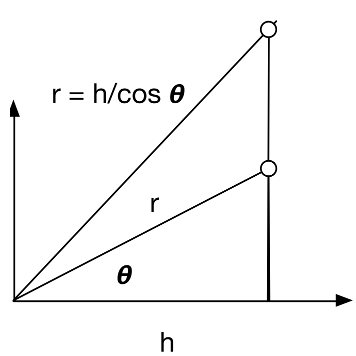
1. Diffuse shading
Diffuse, or Lambertian, shading computation is based on the cosine law for light irradiance.

To understand it, start with the standard trigonometric equation for the cosine $h = r\cos(\theta)$ as on the left. We generally assume that the radius r is held constant, so as θ increases h decreases, as on the right.

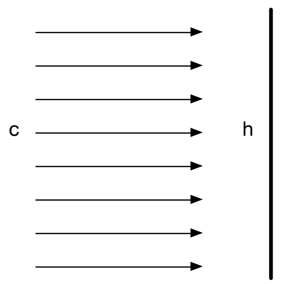




Instead assume h is held constant and r is a function of h, so we have $r = \frac{h}{\cos(\theta)} = h\sec(\theta)$. As θ increases we get r increasing, going to infinite length in the limit as $\theta \rightarrow 90°$

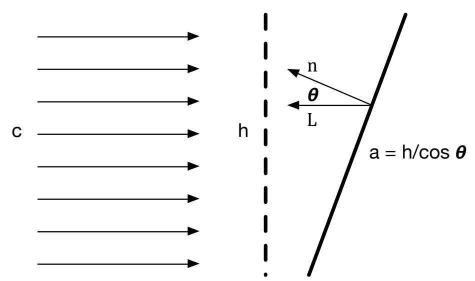


If we reorient our diagram to consider the light from a beam of radiant energy **c** impinging on a surface **h** high we get this diagram when the surface is perpendicular to the incoming light.



Assume for the moment that the surface is 1 unit deep into the page, so the area is **h*1 = h**. Now the irradiance is **c/h**, so the incoming radiant energy is spread over the area h (irradiance is watts over meter squared, or energy over area).

Keep the incoming light the same but tilt the surface and we get this diagram:



Here **a** is the new area over which the incoming radiant energy is spread. **a** increases as the reciprocal of the cos of the tilt angle. We have only a 2D diagram, so **a** is represented here by the length of a line and not properly area, but since we assumed the surface is 1 unit deep into the page and is not tilting into the page, the area increases proportionally to the new length.

Finally, since irradiance = energy/area, we have the new irradiance as $\frac{c}{a} = \frac{c}{h\,/\cos(\theta)} = \frac{c}{h}\cos(\theta)$.

So the new irradiance = old irradiance * cos(θ), which is Lambert's cosine law.

To compute the diffuse shading in a program, we set the irradiance to a constant $c_l = \frac{c}{h}$, and the reflectivity of the surface to k_d. If n is the normal to the surface, and L is the light vector pointing back at the light, and n and L are both normalized to unit length, we have the diffuse contribution as

$$c_d = c_l(\cos(\theta))\,k_d = c_l(\mathrm{n} \bullet \mathrm{L})k_d$$