CMCS427 Notes
Example of determining normal vector for parametric curve: cylinder

Given parametric form for cylinder $p(u,v) = < Rcos(u), h * v, Rsin(u) >$

In this case, to simplify the derivatives the range of u is [0,2π] and the range of v is [0,1]. The y axis is the central axis of the cylinder. The tangent vector to the surface along u is the partial vector derivative wrt u:

$$\frac{dp(u,v)}{du} = < -Rsin(u), 0, Rcos(u) >$$

The tangent vector to the surface along v is the partial vector derivative wrt v:

$$\frac{dp(u,v)}{dv} = < 0, h, 0 >$$

You can see that the partial derivative wrt v is a vertical line, while that wrt u is horizontal and tangent to the circle of the cylinder.

Taking the cross product of the two partial derivatives gives the normal direction.

$$n = \frac{dp(u,v)}{du} \times \frac{dp(u,v)}{dv}$$

For the cylinder we have

$$n = \det\left(\begin{bmatrix} i & j & k \\ -Rsin(u) & 0 & Rcos(u) \\ 0 & h & 0 \end{bmatrix}\right)$$

Which gives

$$n = i\begin{vmatrix} 0 & Rcos(u) \\ h & 0 \end{vmatrix} - j\begin{vmatrix} -Rsin(u) & Rcos(u) \\ 0 & 0 \end{vmatrix} + k\begin{vmatrix} -Rsin(u) & 0 \\ 0 & h \end{vmatrix}$$

$$n =< -hRcos(u), 0, -hRsin(u) >$$

Normalizing n eliminates the hR scale term and we have.

$$\hat{n} =< -cos(u), 0, -sin(u) >$$

Notice that n points into the cylinder. We can negate it to get the outward facing normal. Whether you want dp/du x dp/dv, or dp/dv x dp/du, can depend on how you set up the parameterization.

Readings: http://math.etsu.edu/multicalc/prealpha/Chap3/Chap3-6/printversion.pdf
https://www.khanacademy.org/math/multivariable-calculus/integrating-multivariable-functions/flux-in-3d-articles/a/unit-normal-vector-of-a-surface