Fall 1996 **CMSC 251: Homework 11** Samir Khuller

Due in class: Dec 3.

(1) (a) (5 points) What is the running time of COUNTING-SORT, if we are using it to sort n integers in the range $[a \ldots b]$. Express the running time as a function of n, a and b.

(b) (5 points) Run procedure $PARTITION(A, 1, n)$ on the following array and show the resulting output. Show the exchanges that are done along the way. (Use $A[1]$ as the pivot element.)

7	17	16	3	13	3	11	10	7	6	12
1	2	3	4	5	6	7	8	9	10	11

Figure 1: Input Array

(2) (a) (3 points) Assume that we are given a heap stored in the array A (you may think of the heap as a priority queue). **Draw** a picture of the heap as a rooted tree.

21	17	16	11	13	12	11	10	9	11	12
1	2	3	4	5	6	7	8	9	10	11

Figure 2: Input Array A.

(b) (3 points) **Show** what happens to the heap when we do an INSERT(19) operation on the heap. (This operation adds a new element with key value 19 to the heap.) Show all the exchanges done and not just the resulting tree.

(c) (4 points) **Show** what happens to the heap now when we do an EXTRACT MAX(A) operation from the heap as obtained in part (b). This operation returns the maximum value element and also deletes it from the heap. Show all the exchanges done and not just the resulting tree.

(3) Assume that we are given a heap stored in an array A. The heap currently contains n elements in it, with the maximum element in position $A[1]$. Your job is to write a pseudo-code description of the operation DELETE(A, i). This should run in $O(\log n)$ time. This operation is supposed to delete element $A[i]$ from the heap, and then restore the heap property (in other words, the array A should represent a valid heap of $n - 1$ elements at the end).

You may use procedure $HEAPIFY(A, i)$ without describing its details.

TURN OVER

(4) Consider the following randomized algorithm.

```
X = 0;
i = 1;
while (i ≤ 4) do
    R = RANDOM(0,1) (*this assigns to R a random integer value, either 0 or 1*)
    if R = 1 then X = X + 5
    i = i + 1
end-while
```

It is clear that there will be 4 iterations of the while loop.

Please explain your answers.

(a) What is the probability that the value of X will be 15 when the algorithm terminates?

(b) What is the probability that the value of X will be 0 when the algorithm terminates?

(c) What is the expected value of X upon termination?

(5) Assume that you are given access to function $SELECT(A, n, i)$ that returns the i^{th} smallest element from a set A (of size n) in $O(n)$ time. A contains n (n is odd) distinct elements and is given unsorted. An element has rank i if it is the i^{th} smallest element. You are also given some integer k (not constant!).

(a) (4 points) Write an algorithm to output *all* the elements that have rank $\leq k$ in $O(n)$ time. (You do not have to describe procedure SELECT, you simply need to use it.)

(b) (6 points) Write an algorithm that will find the k elements that are closest in value to the median of the set A in $O(n)$ time.
Example: If set A is 1, 7, 10, 2, 12, 18, 11. Then the median is 10, and the three closest elements (in value) to 10 are 11, 12 and 7. (Hint: Try to construct a new set A' in which the k smallest elements are the elements that you need to output.)