Basic Graph Notation Handout

An undirected graph $G = (V, E)$ is defined as a set V of vertices and a set E of edges. An edge $e = (u, v)$ is an unordered pair of vertices. A directed graph is defined similarly, except that its edges are ordered pairs of vertices, i.e., for a directed graph, $E \subseteq V \times V$. The terms nodes and vertices are used interchangeably. Simple graphs do not have self loops — edges of the form (v, v), or multiple edges connecting two given vertices. When we say "graph" we usually mean "simple graph", unless otherwise specified. The number of vertices of a graph, $|V|$, is often denoted by n. A graph is a sparse graph if $|E| << |V|^2$.

Bipartite graphs form a subclass of graphs and are defined as follows. A graph $G = (V, E)$ is bipartite if the vertex set V can be partitioned into two sets X and Y such that $E \subseteq X \times Y$. In other words, each edge of G connects a vertex in X with a vertex in Y. Such a graph is denoted by $G = (X, Y, E)$. Since bipartite graphs occur commonly in practice, algorithms are often specially designed for them.

A vertex w is adjacent to another vertex v if $(v, w) \in E$. An edge (v, w) is said to be incident to vertices v and w. The neighbors of a vertex v are all vertices $w \in V$ such that $(v, w) \in E$. The number of edges incident to a vertex v is called the degree of vertex v. For a directed graph, if (v, w) is an edge, then we say that the edge goes from v to w. The out-degree of a vertex v is the number of edges from v to other vertices. The in-degree of v is the number of edges from other vertices to v.

A path $p = [v_0, v_1, v_2, \ldots, v_k]$ from v_0 to v_k is a sequence of vertices such that (v_i, v_{i+1}) is an edge in the graph for $0 \leq i < k$. Any edge may be used only once in a path. A cycle is a path whose end vertices are the same, i.e., $v_0 = v_k$. A path is simple if all its internal vertices are distinct. A graph is said to be connected if there is a path between every pair of vertices. A directed graph is said to be strongly connected if there is a path between every pair of vertices in each direction.

An acyclic, undirected graph is a forest, and a tree is a connected forest. A directed graph that does not have any cycles is known as a directed acyclic graph (DAG).

There are two convenient ways of representing graphs on computers. The first is the adjacency list representation. Each vertex has a linked list – there is one entry in the list for each of its adjacent vertices. The graph is thus represented as an array of linked lists, one list for each vertex. This representation uses $O(|V| + |E|)$ storage, which is good for sparse graphs. Such a storage scheme allows one to scan all vertices adjacent to a given vertex in time proportional to the degree of a vertex. The second representation, the adjacency matrix, is as follows. In this scheme, an $n \times n$ array is used to represent the graph. The $[i, j]$ entry of this array is 1 if the graph has an edge between vertices i and j, and 0 otherwise. This representation permits one to test if there is an edge between any pair of vertices in constant time. Both these representation schemes can be used in a natural way to represent directed graphs.