Fall 2001 **CMSC 451: Homework 2** Samir Khuller

Due in class: Sep 27.

Graduate students should do problems 2–6, others do 1-5.

(1) Describe an efficient algorithm that given an undirected graph G, determines a spanning tree of G whose largest edge weight is minimum, over all spanning trees of G. Give an argument justifying your algorithm.

(2) Describe an $O(E+V)$ time algorithm to output the vertices of a graph in topological order. The algorithm should be based on the idea that was discussed in class. Maintain a set Z of vertices with zero in-degree and repeatedly ouput a vertex from Z. What happens if the input graph has a cycle?

(3) A directed graph is said to be semi-connected if, for any two vertices $u, v \in V$ we have that u can reach v or v can reach u. Give an efficient algorithm to determine whether or not G is semi-connected. Prove that your algorithm is correct and analyse its running time.

(4) Show how to implement Prim's algorithm in time $O(|V|^2)$.

(5) Let G be a directed graph. The vertices of G have been numbered $1 \ldots n$ (where n is the number of vertices in G). Let $small(i) = \min\{j | j \text{ is reachable from } i\}$. In other words, for a vertex numbered i, $small(i)$ is the smallest numbered vertex reachable from it. Design an $O(V+E)$ algorithm to compute $small(i)$ for *all* vertices in the graph.

(6) Assume that we have a network (a connected undirected graph) in which each edge e_i has an associated bandwidth b_i. If we have a path P, from s to v, then the capacity of the path is defined to be the minimum bandwidth of all the edges that belong to the path P. We define $capacity(s,v) = \max_{P(s,v)} capacity(P)$. (Essentially, $capacity(s,v)$ is equal to the maximum capacity path from s to v.) Give an efficient algorithm to compute $capacity(s,v)$, for each vertex v; where s is some fixed source vertex. Show that your algorithm is "correct", and analyze its running time.
(Design something that is no more than $O(|V|^2)$, and with the right data structures takes $O(|E| \log |V|)$ time.)