Data Migration on Parallel Disks: Algorithms and Evaluation*

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Abstract. Our work is motivated by the problem of managing data on storage devices, typically a set of disks. Such storage servers are used as web servers or multimedia servers, for handling high demand for data. As the system is running, it needs to dynamically respond to changes in demand for different data items. There are known algorithms for mapping demand to a layout. When the demand changes, a new layout can be computed. In this work we study the *data migration problem*, which arises when we need to quickly change one layout to another. This problem has been studied earlier where for each disk a new layout has been prescribed. However, to apply these algorithms effectively, we identify another problem that we refer to as the correspondence problem, whose solution has a significant impact on the solution for the data migration problem. We study algorithms for the data migration problem in more detail and identify variations of the basic algorithm that seem to improve performance in practice, even though some of the variations have poor worst case behavior.

1 Introduction

To handle high demand, especially for multimedia data, a common approach is to replicate data objects within the storage system. Typically, a large storage server consists of several disks connected using a dedicated network, called a *Storage Area Network*. Disks typically have constraints on space as well as the number of clients that can access data from a single disk simultaneously. The goal is to have the system automatically respond to changes in demand patterns and to recompute data layouts. Such systems and their applications are described and studied in, e.g., [5, 6, 16] and the references therein.

Approximation algorithms have been developed [11–13, 7, 9] to map known demand for data to a specific data layout pattern to maximize utilization. (Utilization refers to the total number of clients that can be assigned to a disk that contains the data they want.) In the layout, we compute not only how many copies of each item we need, but also a layout pattern that specifies the precise subset of items on each disk. (This is not completely accurate and we will elaborate on this step later.) The problem is NP-hard, but there are polynomial time approximation schemes [7, 12, 9]. Given the relative demand for data, an almost optimal layout can be computed.

Over time as the demand patterns change, the system needs to create *new* data layouts. The problem we are interested in is the problem of computing a data migration plan for a set of disks to convert an initial layout to a target layout. We assume that data objects have the same size (these could be data blocks or files) and that it takes the same amount of time to migrate a data item between any pair of

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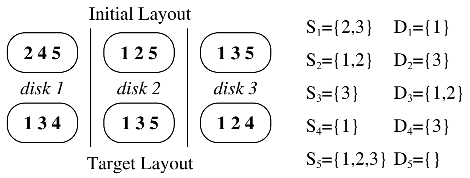

Fig. 1. An initial and target layouts as well as their corresponding S_i's and D_i's. Since item 5 has $D_5 = \emptyset$, we can just drop this item from consideration.

disks. The crucial constraint is that each disk can participate in the transfer of only one item at a time—either as a sender or as a receiver. Our goal is to find a migration schedule to minimize the time taken (i.e., number of rounds) to complete the migration (makespan) since the system is running inefficiently until the new layout has been obtained.

To handle high demand for newly popular objects, new copies will have to be dynamically created and stored on different disks. This means that we crucially need the ability to have a "copy" operation in addition to "move" operations. In previous work [8, 1], the problem is modeled as follows. It is assumed that a specific transfer graph has been given. Each directed edge in the transfer (multi)-graph specifies a move operation from one disk to another. In fact, one of the crucial lower bounds used in the work on data migration [8] is based on a degree property of the multigraph. For example, if the degree of a node is δ, then this is a lower bound on the number of rounds that are required, since in each round at most one transfer operation involving this node may be done. For copying operations, clearly this lower bound is not valid. For example, suppose we have a single copy of a data item on a disk. Suppose we wish to create δ copies of this data item on δ distinct disks. Using the transfer graph approach, we could specify a "copy" operation from the source disk to each of the δ disks. Notice that this would take at least δ rounds. However, by using newly created copies as additional sources we can create δ copies in $\lceil \log(\delta+1) \rceil$ rounds, as in the classic problem of broadcasting by using newly created copies as sources for the data object. (Each copy spawns a new copy in each round.)

One general problem of interest is the **data migration problem with cloning [10]** where data item i resides in a specified (source) subset S_i of disks and needs to be moved to a (destination) subset D_i. In other words, each data item that initially belongs to a subset of disks, needs to be moved to another subset of disks. (We might need to create new copies of this data item and store them on an additional set of disks.) Figure 1 depicts an example. Item 1 resides in disk 2 and disk 3 in the initial layout, and therefore S_1 is $2, 3$. The item needs to be copied to disk 1, as specified in the target layout, and therefore D_1 is 1.

Different communication models can be considered based on how the disks are connected. We use the same model as in [8, 1] where the disks may communicate on any matching; in other words, the underlying communication graph allows for communication between any pair of devices via a matching (e.g., as in a switched storage network with unbounded backplane bandwidth). *This model best captures an architecture of parallel storage devices that are connected on a switched network with sufficient bandwidth. This is most appropriate for our application.* These algorithms can also be extended to models where the size of the matching for each round is constrained [10]. This can be done by a simple simulation, where we only choose a maximal subset of transfers to perform in each round. This model is one of the most widely used in all the work related to gossiping and broadcasting.

1.1 The Correspondence Problem

Given a set of data objects placed on disks, we shall assume that what is important is the grouping of the data objects and not their exact location on each disk. For example, we can represent a disk by the set $\{A, B, C\}$ indicating that data objects A, B, and C are stored on this disk. Thus, modifying the layout of these objects within the same disk does not affect the set corresponding to that disk in any way.

Data layout algorithms (such as the ones in $[11, 12, 9, 7]$) take as input a demand distribution for a set of data objects and output a grouping $S_{1'}, S_{2'}, \ldots S_{N'}$ as a desired data layout pattern on disks $1', 2', \ldots, N'$. (The current layout is assumed to be $S_1, S_2 \ldots S_N$.) Note that we do not need the data corresponding to the set of items $S_{1'}$ to be on (the original) disk 1. For example the algorithm simply requires that a new grouping be obtained where the items in set $S_{1'}$ be grouped together on a disk. For instance, if $S_3 = S_{1'}$ then by simply "renaming" disk 3 as disk $1'$ we have obtained a disk with the set of items $S_{1'}$, assuming that these two disks are inter-changeable. We need to compute a perfect matching between the initial and final layout sets. An edge is present between S_i and $S_{j'}$ if disk i has the same capabilities as disk j'. The weight of this edge is obtained by the number of "new items" that need to be moved to S_i to obtain $S_{j'}$. A minimum weight perfect matching in this graph gives the *correspondence* that minimizes the total number of changes, but *not* the number of rounds. Once we fix the correspondence, we need to invoke an algorithm to compute a migration schedule to minimize the number of rounds. Since this step involves solving an NP-hard problem, we will use a polynomial time approximation algorithm for computing the migration. However, we still need to pick a certain correspondence before we can invoke a data migration algorithm.

There are two central questions in which we are interested; these will be answered in Section 5.4:

- **Which correspondence algorithm should we use?** We will explore algorithms based on computing a matching of minimum total weight and matchings where we minimize the weight of the maximum weight edge. Moreover, the weight function will be based on estimates of how easy or difficult it is to obtain copies of certain data.
- **How good are our data migration algorithms once we fix a certain correspondence?** Even though we have bounds on the worst case performance of the algorithm, we would like to find whether or not its performance is a lot better than the worst case bound. (We do not have any example showing that the bound is tight.) In fact, it is possible that other heuristics perform extremely well, even though they do not have good worst case bounds [10].

Consider the following example. In Figure 2(a) we illustrate a situation where we have 5 disks with the initial and final configurations as shown. By picking the correspondence as shown, we end up with a situation where all the data on the first disk needs to be changed. We have shown the possible edges that can be chosen in the transfer graph along with the labels indicating the data items that we could choose to transfer from a source disk to a destination disk. The final transfer graph shown in Figure 2(a) is a possible output of a data migration algorithm. This will take 5 rounds since all the data is coming to a single disk; that is, node 1 will have a high in-degree. Item V can be obtained from tertiary storage or another device. Clearly, this set of copy operations will be slow and will take many rounds.

On the other hand, if we use the correspondence as shown by the dashed edges in Figure 2(b), we obtain a transfer graph where each disk needs only one new data item and such a transfer can be achieved in two rounds in parallel. (The set of transfers performed by the data migration algorithm are shown in Figure 2(b).)

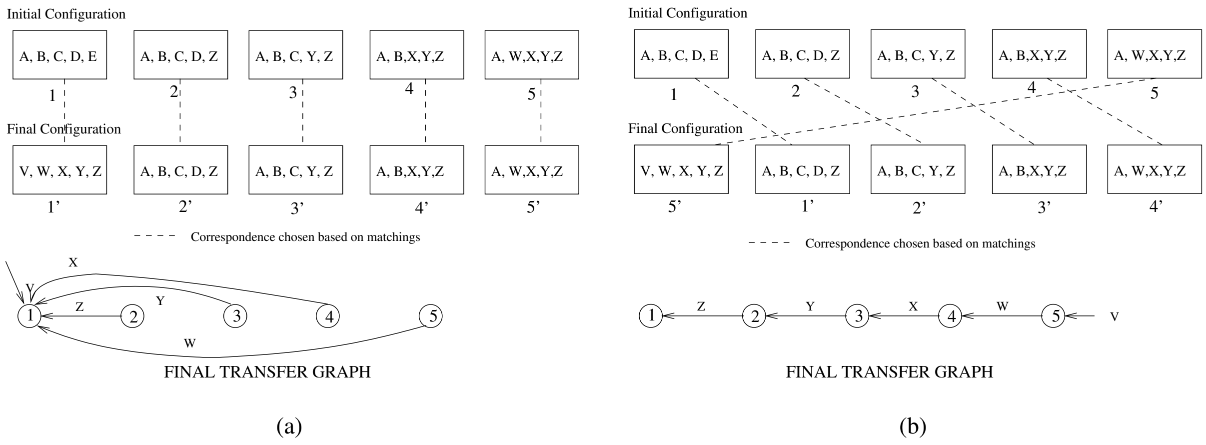

Fig. 2. A bad correspondence can yield a poor solution (the left figure), while a good correspondence can yield significantly better solutions for data movement (the right figure).

1.2 Contributions

In recent work [10], it was shown that the data migration with cloning problem is NP-hard and has a polynomial time approximation algorithm (*KKW*) with a worst case guarantee of 9.5. Moreover, the work also explored a few simple data migration algorithms. Some of these algorithms cannot provide constant approximation guarantees, while for some of the algorithms no approximation guarantee is known. In this paper, we conduct an extensive study of these data migration algorithms' performance under different changes in user access patterns. We find that although the worst-case performance of the KKW algorithm is 9.5, in our experiments the number of rounds required is less than 3.25 times the lower bound. We also identify the correspondence problem and show that a good solution to the correspondence problem can improve the performance of the data migration algorithms by a factor of 4.4 in our experiments, relative to a bad solution. More detailed observations and results of the study are given in Section 5.4.

2 Models and Definitions

In the *data migration problem*, we have N disks and Δ data items. For each item i, there is a subset of disks S_i and D_i. Initially only the disks in S_i have item i, and all disks in D_i want to receive i. Note that after a disk in D_i receives item i, it can be a source of item i for other disks in D_i which have not received the item yet. Our goal is to find a migration schedule using a minimum number of rounds, that is, to minimize the total amount of time to finish the data migration schedule. Our algorithms make use of known results on edge coloring of multigraphs. Given a graph G with max degree Δ_G and multiplicity μ, it is known that it can be colored with at most $\Delta_G + \mu$ colors [15], or at most $1.5\Delta_G$ colors [3].

3 Correspondence Problem Algorithms

To match disks in the initial layout with disks in the target layout, in this paper we consider the following methods, after creating a bipartite graph with two copies of disks:

1. (*Simple min max matching*) The weight of matching disk p in the initial layout with disk q in the target layout is the number of new items that disk q needs to get from other disks (because disks p does not have these items). Then our goal is to find a perfect matching that minimizes the maximum weight of the edges in the matching. Effectively this pairs disks in the initial layout with disks in the target layout, such that the number of items a disk needs to receive is minimized.
2. (*Simple min sum matching*) Minimum weighted perfect matching using the weight function defined in 1. This method minimizes the total number of transfer operations.
3. (*Complex min sum matching*) Minimum weighted perfect matching with another weight function that takes the ease of obtaining an item into account. Note that the larger the ratio of $|D_i|$ to $|S_i|$ the more copying is required. Suppose disk p in the initial layout is matched with disk q in the target layout, and let S be the set of items that disk q needs which are not on disk p. The weight corresponding to matching these two disks is then $\sum_{i \in S} \max(\log \frac{|D_i|}{|S_i|}, 1)$.
4. Direct correspondence. Disk i in the initial layout is always matched with disk i in the target layout.
5. Random permutation.

From our experiments, we found that using a matching-based correspondence method can improve the performance of all data migration algorithms by a factor of 4.4 relative to choosing a bad correspondence. Moreover, we found that all matching-based correspondence methods considered above are comparable to one another, while the Direct correspondence method performs well only when the initial layout and the target layout are similar. Therefore we believe matching-based methods should be used, even though the Direct correspondence and the Random permutation methods run much faster than the matching-based methods. A more detailed description of the results can be found in Section 5.4.

4 Data Migration Algorithms

4.1 KKW Data Migration Algorithm [10]

We now describe KKW data migration algorithm. Define β_j as $|\{i | j \in D_i\}|$, i.e., the number of different sets D_i to which a disk j belongs. We then define β as $\max_{j=1 \ldots N} \beta_j$. In other words, β is an upper bound on the number of items a disk may need. Moreover, we may assume that $D_i \neq \emptyset$ and $D_i \cap S_i = \emptyset$. (We simply define the destination set D_i as the set of disks that need item i and do not currently have it. Thus in Figure 1 item 5 will be dropped.)

The basic idea behind the algorithm is to choose a collection of disjoint subsets $G_i \subseteq D_i$ for each item i. The algorithm then focuses on ensuring that all disks in G_i receive item i. The algorithm then uses the $|G_i| + 1$ copies of the items to deliver them to the set D_i using an edge coloring approach. The algorithm itself is slightly more involved because we need to choose initial sources for each item, without making one disk the source for too many items, and so on. The details can be found in a paper by Khuller, Kim, and Wan [10]. Here is a brief outline of the algorithm:

1. (*Choose sources*) For an item i determine a unique source $s_i \in S_i$ so that $\alpha = \max_{j=1,\ldots,N}(|\{i | j = s_i\}| + \beta_j)$ is minimized. In other words, α is the maximum number of items for which a disk may be a source (s_i) or a destination.
2. (*Large destination sets*) Find a transfer graph for items such that $|D_i| \geq \beta$ as follows.
   (a) (*Choose representatives*) We first compute a disjoint collection of subsets G_i, $i = 1 \ldots \Delta$. Moreover, $G_i \subseteq D_i$ and $|G_i| = \lfloor \frac{|D_i|}{\beta} \rfloor$.
   (b) (*Send to representatives*) We have each item i sent to the set G_i.

(c) (*From representatives to destination sets*) We now create a transfer graph as follows. Each disk is a node in the graph. We add directed edges from disks in G_i to $(\beta - 1)\lfloor \frac{|D_i|}{\beta} \rfloor$ disks in $D_i \setminus G_i$ such that the out-degree of each node in G_i is at most $\beta - 1$ and the in-degree of each node in $D_i \setminus G_i$ is 1. We redefine D_i as a set of $|D_i \setminus G_i| - (\beta - 1)\lfloor \frac{|D_i|}{\beta} \rfloor$ disks which do not receive item i so that they can be taken care of in Step 3. The redefined set D_i has size $< \beta$.

3. (*Small destination sets*) Find a transfer graph for items such that $|D_i| < \beta$ as follows.
   (a) (*Find new sources in small sets*) For each item i, find a new source s'_i in D_i. A disk j can be a source s'_i for several items as long as $\sum_{i \in I_j} |D_i| \leq 2\beta - 1$ where I_j is a set of items of which j is a new source.
   (b) Send each item i from s_i to s'_i.
   (c) Create a transfer graph. We add a directed edge from the new source of item i to all disks in $D_i \setminus \{s'_i\}$.
4. We now find an edge coloring of the transfer graph obtained by merging the two transfer graphs computed in Steps 2(c) and 3(c). The number of colors used is an upper bound on the number of rounds required to ensure that each disk in D_j receives item j.

We now describes several important steps in this algorithm in more detail.

1. Step 1 (*Choose sources*): We find a source $s_i \in S_i$ for each item i so that $\max_{j=1,\ldots,N}(|\{i|j = s_i\}| + \beta_j)$ is minimized, using a flow network as follows. We create a flow network with a source s and a sink t as shown in Figure 3. We have two sets of nodes corresponding to disks and items. Add

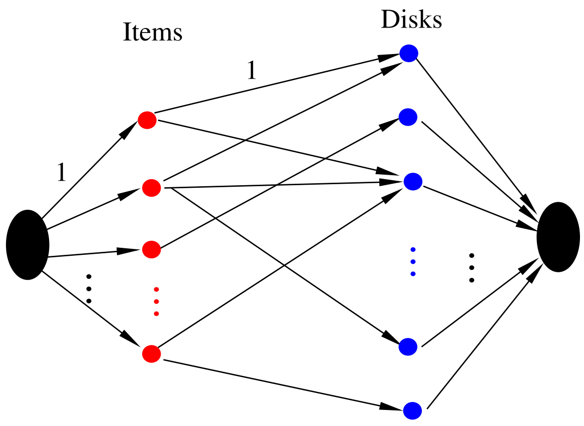

Fig. 3. Flow network to find α

directed edges from s to nodes for items and also directed edges from item i to disk j if $j \in S_i$. The capacities of all those edges are one. Finally we add an edge from the node corresponding to disk j to t with capacity $\alpha - \beta_j$. We want to find the minimum α so that the maximum flow of the network is Δ. We can do this by checking if there is a flow of Δ with α starting from $\max \beta_j$ and increasing by one until it is satisfied. If there is outgoing flow from item i to disk j, then we set j as s_i.
2. Step 2(a) (*Choose representatives*): We choose disjoint sets G_i for each $i = 1 \ldots \Delta$, again using a network flow approach. As shown in Figure 4, we create a flow network with a source s and sink t. In addition we have two sets of vertices U and W. The first set U has Δ nodes, each corresponding to a disk that is the source of an item. The set W has N nodes, each corresponding to a disk in

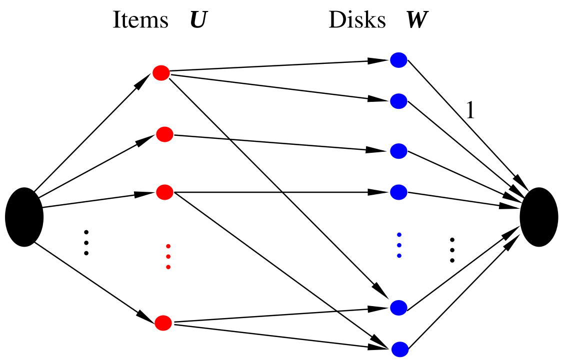

Fig. 4. Flow network to find G_i

the system. We add directed edges from s to each node in U, such that the edge (s, i) has capacity $\lfloor \frac{|D_i|}{\beta} \rfloor$. We also add directed edges with infinite capacity from node $i \in U$ to $j \in W$ if $j \in D_i$. We add unit capacity edges from nodes in W to t. We find a max-flow from s to t in this network. An integral max flow in this network will correspond to $|G_i|$ units of flow going from s to i, and from i to a subset of vertices in D_i before reaching t. The vertices to which i has non-zero flow will form the set G_i.

3. Step 2(b) (*Send to representatives*): A simple solution would be to broadcast the data to each group G_i from the chosen source, since the groups are disjoint. However, this broadcast takes at least $\max_i \log |G_i|$ rounds. Unfortunately, this would give us an $O(\log N)$ approximation guarantee. The method described below, develops stronger lower bounds for this situation.
   Let M be the number of steps required to send all items i to all disks in G_i in an optimal schedule of Step 2(b). To find a lower bound for M, we construct the following flow network F_m (parameterized by an integer m) as shown in Figure 5. We have a source s and two sets of nodes U and V. U has

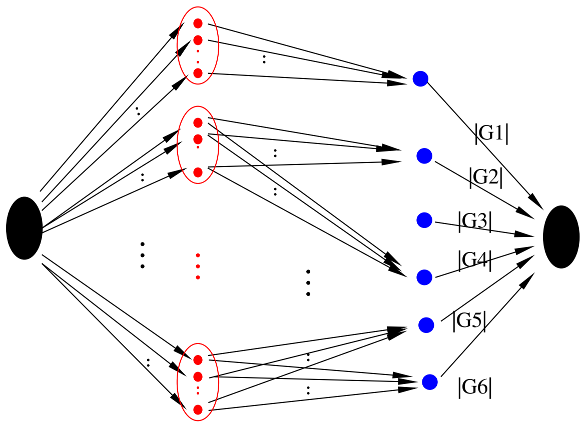

Fig. 5. An example of constructing F_m where $\Delta = 6$

$N \cdot m$ nodes $x_{jk}(j = 1 \dots N, k = 1 \dots m)$. V has Δ nodes $y_i(i = 1 \dots \Delta)$ and y_i has demand $|G_i|$. There is an edge e_{ijk} from x_{jk} to y_i and its capacity c_{ijk} is 2^{m-k} if a disk j has item i initially. There are edges from s to nodes x_{jk} in U with capacity 2^{m-k}. If m' is the smallest number such that we can construct a solution of $F_{m'}$ that satisfies all demands $|G_i|$, then $M \geq m'$.

The solution of the flow network $F_{m'}$ may not correspond to a valid schedule since a node x_{jk} may send flow to several nodes. We then convert this solution to a solution satisfying the following properties.

- node x_{jk} sends flow to at most one node in V;
- the solution satisfies at least $|G_i| - 2^{m'-1}$ demands for each item i.

First, we define a variable z_{ijk} for an edge from x_{jk} to y_i and set $z_{ijk} = f_{ijk}/c_{ijk}$ where f_{ijk} is the flow through e_{ijk} in solution $F_{m'}$. We substitute nodes $y_{il}(l = 1 \ldots \lfloor\sum_{j,k} z_{ijk}\rfloor)$ for each node y_i in V. We distribute edges having nonzero flow to y_i as follows. We then sort edges in non-increasing order of their capacities, and assign edges to y_{i1} until the sum of z values of assigned edges is greater than or equal to one. If the sum is greater than one, then we split the last edge (denoted by $e_{ij'k'}$) into $e_{ij'k'_1}$ and $e_{ij'k'_2}$. We then assign $e_{ij'k'_1}$ to y_{i1} and define $z_{ij'k'_1}$ so that the sum of z values of edges assigned to y_{i1} is exactly one. We set $z_{ij'k'_2} = z_{ij'k'} - z_{ij'k'_1}$, and repeat this so that for all nodes y_{il}, the sums of z values of the assigned edges are one. In the resulting bipartite graph with U and $V' = \{y_{il}\}$, z makes a *fractional* matching which matches all vertices in V'. Therefore, we can find an integral matching that matches all vertices in V' and the matching satisfies the first property in the lemma. Now we merge nodes y_{il} into y_i. Then each y_i matches exactly $\lfloor\sum_{j,k} z_{ijk}\rfloor$ edges.

Now Step 2(b) can be done in $\alpha + 2m' + 1$ rounds based on the above solution. First we choose $\min(\lfloor\sum_{j,k} z_{ijk}\rfloor + 1, |G_i|)$ disks in G_i and denote those disks by H_i. Disk j sends item i to a disk in H_i if edge e_{ijk} is matched for some k. If $|H_i| > \lfloor\sum_{j,k} z_{ijk}\rfloor$, there is one disk in H_i which cannot receive item i. The disk receives item i from s_i. This can be done in $m' + \alpha + 2$ rounds. Since $|G_i|/|H_i| \leq 2^{m'-1}$, we can make all disks in G_i have item i in an additional $m' - 1$ rounds.

4. Step 3(a) (*Find new sources in small sets*): the following result from Shmoys-Tardos [14] will be used for this step.

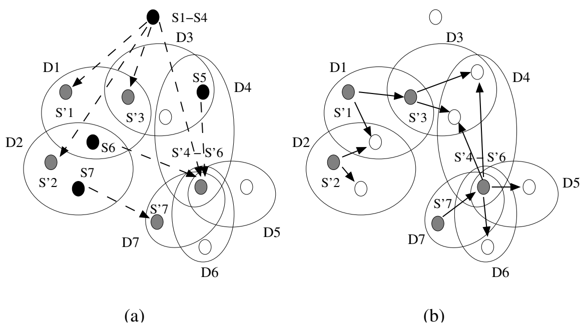

Fig. 6. An example of Step 3 where $\alpha = 4$ and $\beta = 4$. **(a)** migration from s_i to s'_i **(b)** migration from s'_i to $D_i \setminus \{s'_i\}$.

Theorem 1. *(Shmoys-Tardos [14]) We are given a collection of jobs $\mathcal{J}$, each of which is to be assigned to exactly one machine among the set $\mathcal{M}$; if job $j \in \mathcal{J}$ is assigned to machine $i \in \mathcal{M}$, then it requires p_{ij} units of processing time, and incurs a cost c_{ij}. Suppose that there exists a fractional solution (that is, a job can be assigned fractionally to machines) with makespan P and total cost C. Then in polynomial time we can find a schedule with makespan $P + \max p_{ij}$ and total cost C.*

For each item i we wish to choose a source disk s'_i such that the following properties hold (I_j denotes the set of items for which disk j is a source).

- If $i \in I_j$ then $j \in D_i$.
- $\sum_{i \in I_j} |D_i| \leq 2\beta - 1$.

We create an instance of the problem of scheduling machines with costs. Items correspond to jobs and disks correspond to machines. For each item i we define a cost function as follows. $C(i,j) = 1$, if and only if $j \in D_i$; otherwise it is a large constant. Processing time of job i (corresponding to item i) is $|D_i|$ (uniform processing time on all machines). Using Theorem 1 [14], the scheduling algorithm finds a schedule that assigns each job (item) to a machine (disk) to minimize the makespan. Shmoys-Tardos [14] show that the makespan is at most the makespan in a fractional solution plus the processing time of the largest job. Moreover, the cost of that solution is at most the cost of the optimal solution, namely the number of items. We cannot assign an item (job) to a disk (machine), if the disk is not in the destination set for the item.

4.2 KKW Data Migration Algorithm Variants

The 9.5-approximation algorithm for data migration (denoted by *KKW (Basic)*) uses several complicated components to achieve the constant factor approximation guarantee. We consider simpler variants of these components. The variants may not give good theoretical bounds.

(a) in Step 2(a) (*Choose representatives*) we find the minimum integer m such that there exist disjoint sets G_i of size $\lfloor \frac{|D_i|}{m} \rfloor$. The value of m should be between $\bar{\beta} = \sum_{i=1}^{N} \frac{\beta_i}{N}$ and β.
(b) in Step 2(b) (*Send to representatives*) we use a simple doubling method to satisfy all requests in G_i. Since all groups are disjoint, it takes $\max_i \log |G_i|$ rounds.
(c) in Step 3 (*Small destination sets*) we do not find a new source s'_i. Instead s_i sends item i to D_i directly for small sets. We try to find a schedule which minimizes the maximum total degree of disks in the final transfer graph in Step 4.
(d) in Step 3(a) (*Find new sources in small sets*) when we find a new source s'_i, S_is can be candidates as well as D_is. If $s'_i \in S_i$, then we can save some rounds to send item i from s_i to s'_i.

The worst-case time complexity of all of the algorithms, except for variant (c), is $O((n^2+\Delta)n^2\beta \log \frac{(n^2+\Delta)^2}{n^2\beta})$. The worst-case time complexity of variant (c), which does not find new sources s'_i, is $O((n+\Delta)n\Delta \log \frac{(n+\Delta)^2}{n\Delta} \log \Delta)$.

4.3 Heuristic-based Data Migration Algorithms [10]

In this section, we consider several heuristic-based data migration algorithms.

[*Edge Coloring*] We can find a transfer schedule using edge coloring on a transfer graph. Since a disk can get the same item from different source disks, we want to find a good way of selecting source disks. We build a flow network with a source s and a sink t. In addition we have two sets of nodes corresponding to items and source disks. We add edges from s to node i with capacity $|D_i|$ for all item i, and edges from source disks j to t with capacity $c - \beta_j$. Finally, we add edges from item i to source disk j if $j \in S_i$.

Suppose c' is the minimum c such that, for all i, the amount of flow from s to i is the same as its capacity. Such c' can be obtained by binary search and solving a network flow problem in each iteration. Let $f^*(i,j)$ be the flow value from item i to source disk j when c' is obtained. We build a transfer graph as follows. Each disk is a node in the graph. For each source disk j having item i initially, we put $f^*(i,j)$ directed edges from disk j to $f^*(i,j)$ different disks in D_i, meaning that source disk j would send item i to $f^*(i,j)$ disks in D_i. From the flow network, we know that $\sum_{j \in S_i} f^*(i,j) = |D_i|$ for all i. Thus, all destination disks are served. Moreover, each source disk j serves at most $c' - \beta_j$ disks, and receives β_j items from other disks. The total degree of each source disk in the transfer graph is at most c'.

Now we find an edge coloring of the transfer graph (which may be a multigraph) to obtain a valid schedule [3], and the number of colors used is an upper bound on the total number of rounds. The worst-case time complexity here is $O((n + \Delta)n\beta \log \frac{(n+\Delta)^2}{n\beta} + n^2\beta^2)$.

[*Weighted Matching*] The algorithm is as follows.

1. Build a weighted undirected graph as follows. Each disk is a node in the graph. Let the cost $c(i, v, w)$ of sending item i from disk $v \in S_i$ to disk $w \in D_i$ be $\log \frac{|D_i|}{|S_i|}$. Intuitively, if disk v and w are matched, we would like to send the item with greatest cost, and either v or w can be the sender. The weight between v and w in the graph is $\max(\max_i c(i, v, w), \max_i c(i, w, v))$.
2. Find a maximum-weight matching on this graph. For each matched v and w, suppose $c(i, v, w)$ corresponds to the weight between v and w, send item i from v to w. This takes one round to send items in the matched pairs. We then update the S_i's and D_i's.
3. If we have not satisfied all demands (there are some non-empty D_i's), go to Step 1.

Its worst-case time complexity is $O(n^4\beta)$.

[*Unweighted Matching*] The algorithm is the same as *Weighted Matching*, except that the weigth between v and w in the graph is always 1.

[*Item by Item*] In this algorithm, we deal with one item at a time. We process each item i sequentially, and satisfy the demands by doubling the item in each round, which takes $\lceil \log(\frac{D_i}{S_i} + 1) \rceil$. The total number of rounds is $\sum_i \lceil \log(\frac{D_i}{S_i} + 1) \rceil$. The worst-case time complexity here is $O(n\beta)$.

5 Experiments

The framework of our experiments is as follows:

1. (*Create an initial layout*) Run the sliding window algorithm [7], given the number of user requests for each data object. Below we describe the distributions we use in generating user requests. These distributions are completely specified once we fix the ordering of data objects in order of decreasing demand.
2. (*Create a target layout*) To obtain a target layout, we take one of the following approaches.
   (a) Shuffle the ranking of items. Generate a new demand for each item according to the probabilities corresponding to the new ranking of that item. Run the sliding window algorithm again with the new request demands to obtain a target layout.
   (b) Use other (than sliding window) methods to create a target layout. The motivation for exploring these methods is (a) performance issues (as explained later in the paper) as well as (b) that other algorithms (other than sliding window) could perform well in creating layouts. The methods considered here are as follows.

i. Rotation of items: Suppose we numbered the items in non-increasing order of the number of copies in the initial layout. We make a sorted list of items of size $k = \lfloor \frac{\Delta}{50} \rfloor$, and let the list be $l_1, l_2, \ldots, l_k$. Item l_i in the target layout will occupy the position of item l_{i+1} in the initial layout, while item l_k in the target layout will occupy the positions of item l_1 in the initial layout. In other words, the number of copies of items $l_1, \ldots, l_{k-1}$ are decreased slightly, while the number of copies of item l_k is increased significantly.

ii. Enlarging D_i for items with small S_i: Repeat the following $\lfloor \frac{\Delta}{20} \rfloor$ times. Pick an item s randomly having only one copy in the current layout. For each item i that has more than one copy in the current layout, there is a probability of 0.5 that item i will randomly give up the space of one of its copies, and the space will be allocated to item s in the new layout for the next iteration. In other words, if there are k items having more than one copy at the beginning of this iteration, then item s is expected to gain $\frac{k}{2}$ copies at the end of the iteration.

3. (*Find a correspondence*) Run different correspondence algorithms given in Section 3 to match a disk in the initial layout with a disk in the target layout. Now we can find the set of source and destination disks for each item.
4. (*Compute a migration schedule*) Run different data migration algorithms, and record the number of rounds needed to finish the migration.

5.1 User Request Distributions

We generate the number of requests for different data objects using Zipf distributions and Geometric distributions. We note that few large-scale measurement studies exist for the applications of interest here (e.g., video-on-demand systems), and hence below we are considering several potentially interesting distributions. Some of these correspond to existing measurement studies (as noted below) and others we consider in order to explore the performance characteristics of our algorithms and to further improve the understanding of such algorithms. For instance, a Zipf distribution is often used for characterizing people's preferences.

Zipf Distribution The Zipf distribution is defined as follows:

$$\text{Prob(request for movie } i) = \frac{c}{i^{1-\theta}} \quad \begin{array}{c} \forall i = 1, \ldots, M \\ \text{and} \\ 0 \leq \theta \leq 1 \end{array}$$

$$\text{where} \quad c = \frac{1}{H_M^{1-\theta}} \quad \text{and} \quad H_M^{1-\theta} = \sum_{j=1}^{M} \frac{1}{j^{1-\theta}}$$

and θ determines the degree of skewness. For instance, $\theta = 1.0$ corresponds to the uniform distribution, whereas $\theta = 0.0$ corresponds to the skewness in access patterns often attributed to movies-on-demand type applications, e.g., similar to the *measurements* performed in [4]. We assign θ to be 0 and 0.5 in our experiments below.

Geometric Distribution We also tried Geometric Distributions in order to investigate how a more skewed distribution affects the performance of the data migration algorithms. The distribution is defined as follows:

$$\text{Prob(request for movie } i) = (1-p)^{i-1} p \quad \begin{array}{c} \forall i = 1, \ldots, M \\ \text{and} \\ 0 < p < 1 \end{array}$$

where we use p set to 0.25 and 0.5 in our experiments below.

5.2 Shuffling methods

We use the following shuffling methods in Step 2(a) in the beginning of Section 5.

1. Randomly promote 20% of the items. For each chosen item of rank i, we promote it to rank 1 to $i-1$, randomly.
2. Promote the least popular item to the top, and demote all other items by one rank.

5.3 Parameters and Layout Creation

We now describe the parameters used in the experiments, namely the number of disks, space capacity, and load capacity. We ran a number of experiments with 60 disks. For each correspondence method, user request distribution, and shuffling method, we generated 20 inputs (i.e., 20 sets of initial and target layouts) for each set of parameters, and ran different data migration algorithms on those instances. In the Zipf distribution, we used θ values of 0 and 0.5, and in the Geometric distribution, we assigned p values of 0.25 and 0.5.

We tried three different pairs of settings for space and load capacities, namely: (A) 15 and 40, (B) 30 and 35, and (C) 60 and 150. We obtained these numbers from the specifications of modern SCSI hard drives. For example, a 72GB 15,000 rpm disk can support a sustained transfer rate of 75MB/s with an average seek time of around 3.5ms. Considering MPEG-2 movies of 2 hours each with encoding rates of 6Mbps, and assuming the transfer rate under parallel load is 40% of the sustained rate, the disk can store 15 movies and support 40 streams. The space capacity 30 and the load capacity 35 are obtained from using a 150GB 10,000 rpm disk with a 72MB/s sustained transfer rate. The space capacity 60 and the load capacity 150 are obtained by assuming that movies are encoded using MPEG-4 format (instead of MPEG-2). So a disk is capable of storing more movies and supporting more streams.

We show the results of 5 different layout creation schemes with different combinations of methods and parameters to create the initial and target layouts. (I): Promoting the last item to the top, Zipf distribution ($\theta = 0$); (II): Promoting 20% of items, Zipf distribution ($\theta = 0$); (III): Promoting the last item to the top, Geometric distribution ($p = 0.5$); (IV): Initial layout obtained from the Zipf distribution ($\theta = 0$), target layout obtained from the method described in Step 2(b)i in the beginning of Section 5 (rotation of items); and (V): Initial layout obtained from the Zipf distribution ($\theta = 0$), target layout obtained from the method described in Step 2(b)ii in the beginning of Section 5 (enlarging D_i for items with small S_i). We also tried promoting 20% of items using the Geometric distribution. The details of this result are omitted since they were qualitatively similar to those presented below.

5.4 Results

In the tables we present the average for 20 inputs. Moreover, we present results of two representative inputs individually, to illustrate the performance of the algorithms under the same initial and target layouts. This presentation is motivated as follows. The *absolute* performance of each run is largely a function of the differences between the initial and the target layouts (and this is true for all algorithms). That is, a small difference is likely to result in relatively few rounds needed for data migration, and a large difference is likely to result in relatively many rounds needed for data migration. Since a goal of this study is to understand the *relative* performance differences between the algorithms described above, i.e., given the same initial and target layouts, we believe that presenting the data on a per run basis is more informative. That is, considering the average alone somewhat obscures the characteristics of the different algorithms.

Different correspondence methods We first investigate how different correspondence methods affect the performance of the data migration algorithms. Figures 7 and 8 show the ratio of the number of rounds taken by different data migration algorithms to the lower bounds, averaged over 20 inputs under parameter setting (A). We observed that the simple min max matching (1) always returns the same matching as the simple min sum matching (2) in all instances we tried. Moreover, using a simpler weight function (2) or a more involved one (3) does not seem to affect the behavior in any significant way (often these matchings are the same). Thus we only present results using simple min max matching, and label this as matching-based correspondence method.

From the figures, we found that using a matching-based method is important and can affect the performance of all algorithms by up to a factor of 4.4 in our experiments as compared to a bad correspondence, using a Random permutation for example. Since Direct correspondence does not perform as well as other weight-based matchings, this also suggests that a good correspondence method is important. However, the performance of Direct correspondence was reasonable when we promoted the popularity of one item. This can be explained by the fact that in this case sliding window obtains a target layout which is fairly similar to the initial layout.

Different data migration algorithms From the previous section it is reasonable to evaluate the performance of different data migration algorithms using only the simple min max matching correspondence method. We now compare the performance of different variants of the KKW algorithm. Consider algorithm KKW (c) which modifies Step 3 (where we want to satisfy small D_i); we found that sending the items from S_i to small D_i directly using edge coloring, without using new sources s_i', is a much better idea. Even though this makes the algorithm an $O(\Delta)$ approximation algorithm, the performance is very good under both the Zipf and the Geometric distributions, since the sources are not concentrated on one disk and only a few items require rapid doubling.

In addition, we thought that making the sets G_i slightly larger by using $\overline{\beta}$ was a better idea (i.e., algorithm KKW (a) which modifies Step 2(a)). This reduces the average degree of nodes in later steps such as in Step 2(c) and Step 3 where we send the item to disks in $D_i \setminus G_i$. However, the experiments show that it usually performs slightly worse than the algorithm using β.

Consider algorithm KKW (d) which modifies Step 3(a) (where we identify new sources s_i'): we found that the performance of the variant that includes S_i, in addition to D_i, as candidates for the new source s_i' is mixed. Sometimes it is better than the basic KKW algorithm, but more often it is worse.

Consider algorithm KKW (b) which modifies Step 2(b) (where we send the items from the sources S_i to G_i); we found that doing a simple broadcast is generally a better idea, as we can see from the results for Parameter Setting (A), Instance 1 in Table 4 and for Parameter Setting (A), Instance 1 in Table 5. Even though this makes the algorithm an $O(\log n)$ approximation algorithm, very rarely is the size of $\max G_i$ large. Under the input generated by Zipf and Geometric distributions, the simple broadcast generally performs the same as the basic KKW algorithm since the size of $\max G_i$ is very small.

Out of all the heuristics, the matching-based heuristics perform very well in practice. The only cases where they perform extremely badly correspond to hand-crafted (by us) bad examples. Suppose, for example, a set of Δ disks are the sources for Δ items (each disk has all Δ items). Suppose further that the destination disks also have size Δ each and are disjoint. The results are given in Figure 10 and Table 1. The KKW algorithm sends each of the items to one disk in D_i in the very first round. After that, a broadcast can be done in the destination sets as they are disjoint, which takes $O(\log \Delta)$ rounds in total. Therefore the ratio of the number of rounds used to the lower bound remains a constant. The matching-based algorithm can take up to Δ rounds, as it can focus on sending item i at each round by computing a perfect matching of size Δ between the source disks and the destination disks for item i in each round. Since any perfect matching costs the same weight in this case, the matching focuses on sending only one item in each round. We implemented a variant of the weighted matching heuristic to get around this problem by adding a very small random weight to each edge in the graph. As we can see from Figure 10 and Table 1, although this variant does not perform as well as the KKW algorithm, it performs much better than other matching-based data migration algorithms. Moreover, we ran this variant with the inputs generated from Zipf and Geometric distributions, and we found that it frequently takes the same number of rounds as the weighted matching heuristic. In some cases it performs better than the weighted matching heuristic, while in a few cases its performance is worse.

Given the performance of different data migration algorithms illustrated in Figure 9 and in all the tables, the two matching-based heuristics are comparable. Matching-based heuristics perform the best in general, followed by the edge-coloring heuristic, followed by the KKW algorithms, while processing items one-by-one comes last. The main reason why the edge-coloring heuristic performs better than the basic KKW algorithm is that the input contains mostly move operations, i.e., the sizes of S_i and D_i are at most 2 for at least 80% of the items. The Zipf distribution does not provide enough cloning operations for the KKW algorithm to show its true potential (the sizes of G_i are mostly zero, with one or two items having sizes of 1 or 2). Processing items one by one is bad because we have a lot of items (more than 300 in Parameter Setting (A)), but most of the operations can be done in parallel (we have 60 disks, meaning that we can support 30 copy operations in one round). Under the Zipf distribution, since the sizes of most sets G_i are zero, the data migration variant which sends items from S_i directly to D_i is essentially the same as the coloring heuristic. Thus they have almost the same performance.

Under different input parameters In additional to the Zipf distribution, we also tried the Geometric distribution because we would like to investigate the performance of the algorithms under more skewed distributions where more cloning of items is necessary. As we can see in Figure 8 and Table 4, we found that the performance of the coloring heuristic is worse than the KKW algorithms, especially when p is large (more skewed) or when the ratio of the load capacity to space capacity is high. However, the matching-based heuristics still perform the best.

We also investigated the effect of a higher ratio of the load to space capacities on the performance of different algorithms. We found the results to be qualitatively similar, and thus we omit them here.

Moreover, in the Zipf distribution, we assigned different values to θ, which controls the skewness of the distribution; specifically, we considered values of 0.0 and 0.5. We found that the results are similar in both cases. While in the Geometric distribution, a higher value of p (0.5 vs 0.25) gives the KKW algorithms an advantage over coloring heuristic as more cloning is necessary.

Miscellaneous Tables 5 and 6 show the performance of different algorithms using inputs where the target layout is derived from the initial layout, as described in Step 2(b)i and Step 2(b)ii in Section 5. Note that when the initial and target layouts are very similar, the data migration can be done very quickly. The number of rounds taken is much fewer than the number of rounds taken using inputs generated from running the sliding window algorithm again to create the target layout. This illustrates that it may be worthwhile to consider developing an algorithm which takes in an initial layout and the new access pattern, and then derives a target layout, with the optimization of the data migration process in mind. Developing these types of algorithms and evaluating their performance characteristics are part of our future efforts.

Tables 2 and 3 show the performance of different algorithms under two shuffling methods described in Section 5.2. We found that the results are qualitatively similar.

We now consider the running time of the different data migration algorithms. Except for the matching heuristics, all other algorithms' running times are at most 3 CPU seconds and often less than 1 CPU second, on a Sun Fire V880 server. The running time of the matching heuristics depends on the total number of items. It can be as high as 43 CPU seconds when the number of items is around 3500, and it can be lower than 2 CPU seconds when the number of items is around 500.

We also collected the maximum space requirements for each disk needed to complete the migration. Consider disk 3 in Figure 1 and suppose that another disk needs item 3 from disk 3. If disk 3 receives items 2 and 4 before sending out item 3, then its maximum temporary space requirement is 4. However, since all data migration algorithms listed in this paper do not optimize for the maximum temporary space requirement, in many instances, there exists a disk that needs twice the original space requirements.

Final Conclusions For the correspondence problem question posed in Section 1.1, our experiments indicate that weighted matching is the best approach among the ones we tried.

For the data migration problem question posed in Section 1.1, our experiments indicate that the weighted matching heuristic with some randomness does very well. This suggests that perhaps a variation of matching can be used to obtain an $O(1)$ approximation as well. Among all variants of the KKW algorithm, letting S_i send item i to D_i directly for the small sets, i.e., variant (c), performs the best. From the above described results we can conclude that under the Zipf and Geometric distributions, where cloning does not occur frequently, the weighted matching heuristic returns a schedule which requires only a few rounds more than the optimal. All variants of the KKW algorithm usually take no greater than 10 more rounds than the optimal, when a good correspondence method is used.

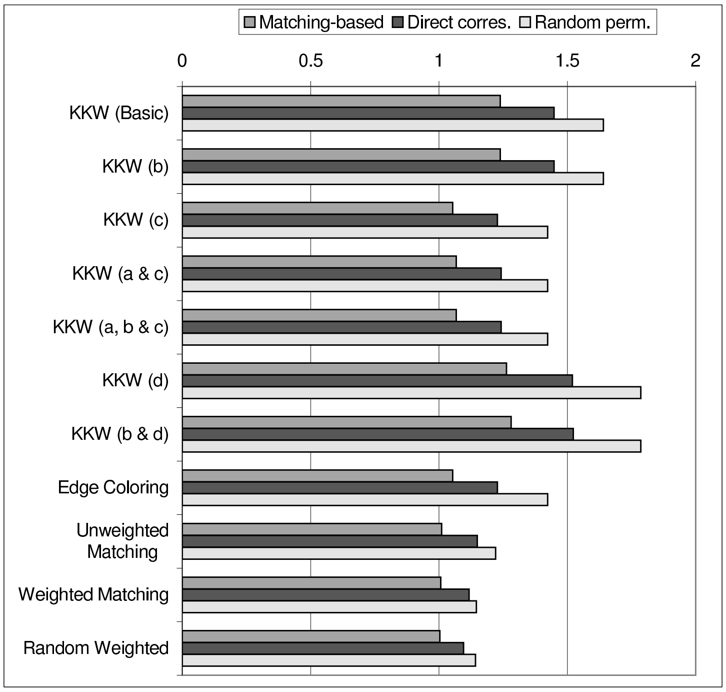

Fig. 7. The ratio of the number of rounds taken by the algorithms to the lower bound (28.1 rounds), averaged over 20 inputs, using parameter setting (A) and layout creation scheme (I) (i.e., with 60 disks, space cap of 15, load cap of 40, promoting the last item to the top, and user requests following the Zipf distribution ($\theta = 0$)). The effect under different correspondence methods is shown.

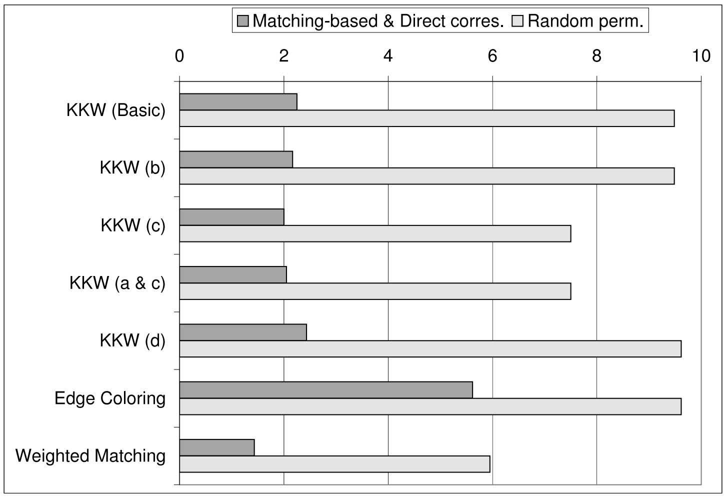

Fig. 8. The ratio of the number of rounds taken by the algorithms to the lower bound (6.0 rounds), averaged over 20 inputs, using parameter setting (A) and layout creation scheme (II) (i.e., with 60 disks, space cap of 15, load cap of 40, promoting the last item to the top, and user requests following the Geometric distribution ($p = 0.5$)). The effect under different correspondence methods is shown.

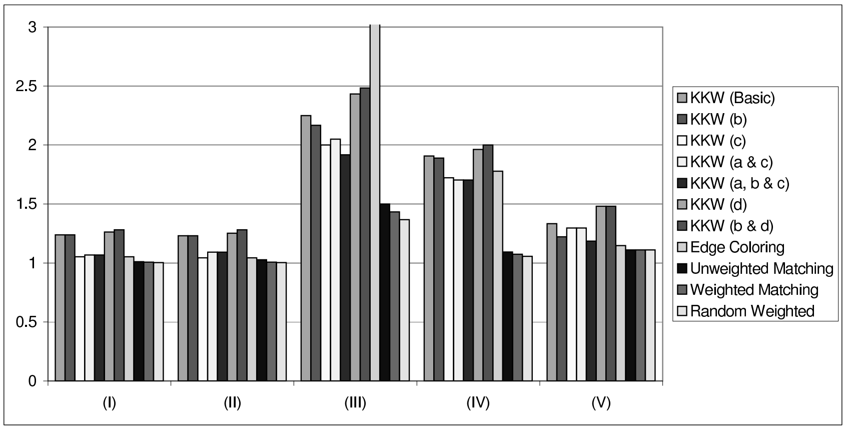

Fig. 9. The ratio of the number of rounds taken by the algorithms to the lower bound, averaged over 20 inputs, using min max matching correspondence method and parameter setting (A), under different layout creation schemes (see Section 5.3). Note that the order of the bars in each cluster is the same as that in the legend.

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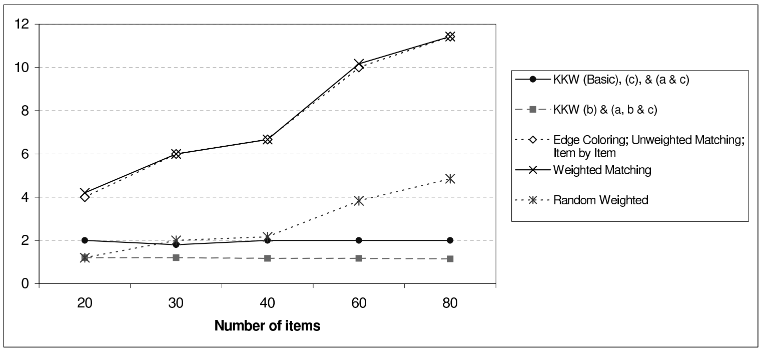

Fig. 10. The ratio of the number of rounds taken by the algorithms to the lower bound under the worst case.

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Table 1. The number of rounds taken by different data migration algorithms, when a set of Δ disks are the sources for Δ items (each disk has all Δ items), and the destination disks also have size Δ each and are disjoint.

Number of items (Δ):	20	30	40	60	80
Lower Bound	5	5	6	6	7
KKW (Basic)	10	9	12	12	14
KKW ((b) doubling)	6	6	7	7	8
KKW ((c) S_i to D_i)	10	9	12	12	14
KKW ((a) $\bar{\beta}$, (c) S_i to D_i)	10	9	12	12	14
Edge Coloring	20	30	40	60	80
Unweighted Matching	20	30	40	60	80
Weighted Matching	21	30	40	61	80
Random Weighted	6	10	13	23	34
Item by Item	20	30	40	60	80

Table 2. The ratio of the number of rounds taken by the data migration algorithms to the lower bounds, using min max matching correspondence method with layout creation scheme (I) (i.e., promoting the last item to the top, with user requests following the Zipf distribution ($\theta = 0$)), and under different parameter settings.

Parameter setting:	(A)			(B)			(C)		
Instance:	1	2	Ave	1	2	Ave	1	2	Ave
KKW (Basic)	1.233	1.233	1.238	1.130	1.104	1.122	1.055	1.108	1.096
KKW (b)	1.233	1.233	1.238	1.130	1.104	1.122	1.055	1.108	1.096
KKW (c)	1.000	1.033	1.053	1.000	1.000	1.006	1.000	1.092	1.044
KKW (a & c)	1.033	1.067	1.068	1.022	1.021	1.021	1.000	1.092	1.044
KKW (a, b & c)	1.033	1.067	1.068	1.022	1.021	1.021	1.000	1.092	1.044
KKW (d)	1.000	1.300	1.263	1.000	1.000	1.014	1.191	1.292	1.169
KKW (b & d)	1.133	1.167	1.281	1.022	1.021	1.037	1.191	1.292	1.170
Edge Coloring	1.000	1.033	1.053	1.000	1.000	1.006	1.000	1.092	1.044
Unweighted Matching	1.000	1.000	1.011	1.000	1.000	1.000	1.000	1.031	1.009
Weighted Matching	1.000	1.000	1.007	1.000	1.000	1.000	1.000	1.015	1.006
Random Weighted	1.000	1.000	1.004	1.000	1.000	1.000	1.000	1.015	1.008
Item by Item	11.100	10.567	12.313	6.522	7.188	7.822	8.455	7.646	7.654

Table 3. The ratio of the number of rounds taken by the data migration algorithms to the lower bounds, using min max matching correspondence method with layout creation scheme (II) (i.e., promoting 20% of items, with user requests following the Zipf distribution ($\theta = 0$)), and under different parameter settings.

Parameter setting:	(A)			(B)			(C)		
Instance:	1	2	Ave	1	2	Ave	1	2	Ave
KKW (Basic)	1.267	1.233	1.231	1.100	1.121	1.092	1.042	1.059	1.048
KKW (b)	1.267	1.233	1.231	1.100	1.121	1.092	1.042	1.059	1.048
KKW (c)	1.033	1.033	1.044	1.000	1.017	1.008	1.008	1.008	1.013
KKW (a & c)	1.067	1.300	1.092	1.000	1.017	1.008	1.008	1.008	1.013
KKW (a, b & c)	1.067	1.300	1.092	1.000	1.017	1.008	1.008	1.008	1.013
KKW (d)	1.367	1.167	1.252	1.167	1.207	1.149	1.203	1.263	1.201
KKW (b & d)	1.367	1.267	1.282	1.167	1.207	1.149	1.203	1.263	1.201
Edge Coloring	1.033	1.033	1.044	1.000	1.017	1.008	1.008	1.008	1.013
Unweighted Matching	1.067	1.000	1.027	1.033	1.034	1.023	1.017	1.025	1.015
Weighted Matching	1.033	1.000	1.007	1.033	1.017	1.003	1.008	1.000	1.003
Random Weighted	1.000	1.000	1.003	1.000	1.017	1.002	1.000	1.000	1.000
Item by Item	16.867	16.067	16.639	14.283	15.345	14.072	15.839	15.686	15.566

Table 4. The ratio of the number of rounds taken by the data migration algorithms to the lower bounds, using min max matching correspondence method with layout creation scheme (III) (i.e., promoting the last item to the top, with user requests following the Geometric distribution ($p = 0.5$)), and under different parameter settings.

Parameter setting:	(A)			(B)		
Instance:	1	2	Ave	1	2	Ave
KKW (Basic)	2.000	2.167	2.250	1.611	1.611	1.568
KKW (b)	1.875	2.167	2.167	1.611	1.611	1.568
KKW (c)	1.625	2.000	2.000	1.444	1.222	1.286
KKW (a & c)	1.750	2.000	2.050	1.333	1.333	1.296
KKW (a, b & c)	1.625	2.000	1.917	1.278	1.278	1.261
KKW (d)	1.750	2.333	2.433	1.722	1.500	1.533
KKW (b & d)	1.875	2.333	2.483	1.556	1.556	1.538
Edge Coloring	3.875	5.667	5.617	2.000	2.111	1.980
Unweighted Matching	1.000	1.500	1.500	1.111	1.111	1.116
Weighted Matching	1.000	1.500	1.433	1.056	1.111	1.111
Random Weighted	1.000	1.333	1.367	1.056	1.056	1.010
Item by Item	6.250	12.833	12.683	3.667	3.667	5.719

Table 5. The ratio of the number of rounds taken by the data migration algorithms to the lower bounds, using min max matching correspondence method with layout creation scheme (IV) (i.e., target layout obtained from the method described in Step 2(b)i in Section 5 (rotation of items)), and under different parameter settings.

Parameter setting:	(A)			(B)			(C)		
Instance:	1	2	Ave	1	2	Ave	1	2	Ave
KKW (Basic)	2.400	2.000	1.907	1.417	1.444	1.553	1.333	1.250	1.330
KKW (b)	2.200	2.000	1.889	1.417	1.444	1.553	1.333	1.250	1.330
KKW (c)	2.000	1.600	1.722	1.167	1.111	1.289	1.222	1.000	1.107
KKW (a & c)	1.800	2.000	1.704	1.250	1.333	1.408	1.222	1.083	1.170
KKW (a, b & c)	2.000	2.000	1.704	1.333	1.333	1.408	1.222	1.083	1.170
KKW (d)	2.400	2.400	1.963	1.333	1.444	1.513	1.556	1.083	1.348
KKW (b & d)	2.200	2.200	2.000	1.250	1.667	1.658	1.556	1.333	1.393
Edge Coloring	1.800	2.000	1.778	1.000	1.000	1.250	1.222	1.000	1.125
Unweighted Matching	1.000	1.000	1.093	1.000	1.000	1.026	1.000	1.000	1.000
Weighted Matching	1.000	1.200	1.074	1.000	1.000	1.013	1.000	1.000	1.009
Random Weighted	1.000	1.200	1.056	1.000	1.000	1.013	1.000	1.000	1.009
Item by Item	10.000	9.800	8.889	6.333	8.111	9.592	15.778	12.000	12.536

Table 6. The ratio of the number of rounds taken by the data migration algorithms to the lower bounds, using min max matching correspondence method with layout creation scheme (V) (i.e., target layout obtained from the method described in Step 2(b)ii in Section 5 (enlarging D_i for items with small S_i)), and under different parameter settings.

Parameter setting:	(A)			(B)			(C)		
Instance:	1	2	Ave	1	2	Ave	1	2	Ave
KKW (Basic)	1.000	1.333	1.333	1.000	1.000	1.114	1.167	1.000	1.140
KKW (b)	1.000	1.333	1.222	1.000	1.000	1.114	1.167	1.000	1.140
KKW (c)	1.000	1.333	1.296	1.000	1.000	1.086	1.000	1.000	1.093
KKW (a & c)	1.000	1.333	1.296	1.000	1.000	1.086	1.167	1.000	1.116
KKW (a, b & c)	1.000	1.333	1.185	1.000	1.000	1.086	1.167	1.000	1.093
KKW (d)	1.000	1.667	1.481	1.000	1.500	1.286	1.500	1.750	1.488
KKW (b & d)	1.000	1.667	1.481	1.000	1.500	1.286	1.667	1.750	1.512
Edge Coloring	1.000	1.000	1.148	1.000	1.000	1.057	1.000	1.000	1.047
Unweighted Matching	1.000	1.000	1.111	1.000	1.000	1.029	1.000	1.000	1.047
Weighted Matching	1.000	1.000	1.111	1.000	1.000	1.029	1.000	1.000	1.047
Random Weighted	1.000	1.000	1.111	1.000	1.000	1.029	1.000	1.000	1.047
Item by Item	7.000	5.000	5.815	15.000	7.750	8.200	8.833	11.750	11.349