
Total points: 30. Total time: 115 minutes. 4 problems over 4 pages. No book, notes, or calculator

1. [10 points]

Suppose Bob uses RSA with $n=77$ and $e=5$.

Are these valid numbers for RSA. Explain.

If you answer yes, obtain the corresponding d .

2. [5 points]

Assume that Bob uses RSA and the following hold:

- $n=15$
- Bob's signature of message $m=2$ is 5
- Bob's signature of message $m=3$ is 4

Obtain Bob's signature for the message $m=12$. Show your derivation here.

3. [5 points]

How many numbers between 1 and 250000 are relatively prime to 250000? Explain

4. [10 points]

Using the efficient algorithm, compute $131^{25} \bmod 15$

5. [10 points]

Obtain a formula that yields a number x in \mathbb{Z}_{45} such that $x \bmod 5 = x_1$ and $x \bmod 9 = x_2$.

Or if you think such a formula does not exist, explain.