Randomized Algorithms and Probabilistic Analysis in Wireless Networking*

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Abstract. Devices connected wirelessly, in various forms including computers, hand-held devices, *ad hoc* networks, and embedded systems, are expected to become ubiquitous all around us. Wireless networks pose interesting new challenges, some of which do not arise in standard (wired) networks. This survey discusses some key probabilistic notions – both randomized algorithms and probabilistic analysis – in wireless networking.

1 Introduction

It is anticipated that wireless networking will continue to have considerable growth in the foreseeable future, and that devices connected in wireless fashion will pervade our world. As compared to wired networks, two particular challenges arise in the wireless setting: *energy-conservation* (since tiny and embedded devices, especially, have very little access to a continuous source of power) and *interference* between nearby transmissions. This survey will briefly reference certain algorithmic approaches and modeling/analysis techniques that have been developed to tackle such issues. This is certainly not meant to be an encyclopedic survey, and many key papers will not be referred to here. Rather, we hope to spur the interest of the reader in exploring this exciting area further.

It is natural that probabilistic considerations should help in our present context. First, in addition to their well-known advantages (such as leading to faster and simpler algorithms), randomized algorithms play a powerful role in any type of *distributed* system, through paradigms such as symmetry-breaking; a natural example of this is contention resolution at the MAC (Media Access Control) layer of wireless networks, where nearby radios try to access their local radio spectrum in a contention-free manner for short periods of time. Second, probabilistic analysis is useful as always in determining "typical" properties of systems. Consider the example of a set of n transceivers distributed randomly in a bounded region: this is a particularly natural way to model a set of n sensors thrown over the region. Probabilistic models are also one obvious approach to model mobility.

We will reference below a few representative examples of randomized algorithms and probabilistic analysis in wireless networking. The reader is referred to [22] for a comprehensive tutorial on algorithms for sensor networks.

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2 Four topics

We briefly consider four topics: (a) energy conservation, (b) design and analysis of media-access protocols, (c) probabilistic analysis of network capacity assuming a random network topology and/or a random traffic matrix, and (d) probabilistic analysis/randomized algorithms in efficient aggregation of information in sensor networks.

A candidate approach that has been proposed for energy savings is through co-operation: subsets of nodes act as a *backbone* while other nodes go to sleep, and the backbone stores and forwards messages for all nodes during this period [5]. This backbone is updated frequently, in the interest of fairness. Graph-theoretically, such problems are closely related to connected domination (which is a natural condition to impose on the backbone), domatic partitions (the problem of partitioning the network into backbones) etc. Randomization plays an essential role in much work in this area; see, e.g., [5, 8, 6]. However, good *deterministic* deterministic approximation algorithms are also possible, in the case where the underlying network satisfies inter-node distances that approximate those in some low-dimensional Euclidean space (e.g., if the network has small *doubling-dimension*) [19]. However, what if the nodes are selfish and will deviate from the protocol if it satisfies their individual selfish desire to conserve their own power? See [16] for a game-theoretic approach to this problem.

Second, as mentioned above, random access is a natural approach for accessing the medium (i.e., the radio spectrum). While this is a classical, well-studied issue (see, e.g., [1, 3, 11]), the analysis becomes much harder for modern protocols in wireless networking; this is due to additional constraints and features such as routing in the network, the possibility of multi-channel multi-radio transceivers, etc. The works [21, 12] respectively analyze protocols such as relatives of IEEE 802.11 and present new random-access protocols for multihop wireless networks. Combined MAC scheduling and end-to-end routing for a given set of end-to-end connections is achieved in [14]; the ideas include linear-programming relaxations, distributed graph-coloring, and geometric arguments. See [4] for a provably-good deterministic distributed packet-scheduling algorithm for the multi-channel multi-radio setting.

Our next two topics largely involve probabilistic analysis.

Our third topic relates to the fact that as opposed to wired networks, the capacity or throughput (measured using maximum flow, maximum concurrent flow, or other similar objective functions) of wireless networks is complicated by the presence of interference. In a seminal paper, Gupta & Kumar considered the capacity of a multi-hop wireless network formed by distributed n transceivers randomly in the unit square; the traffic matrix is random (such as a random permutation of the transceivers), and the interference model can be any one of a few standard candidates [10]. This spurred quite some work on generalizations (e.g., to hybrid networks which have a few base stations, see [17, 13]). The capacity has also been approximated in the worst case using geometric arguments [15]. Random-graph models for wireless (sensor) networks have been analyzed in works including [20, 9, 18].

Finally, we very briefly mention the field of collecting/aggregating information from the sensors in a sensor network, in an energy-efficient manner. The basic idea is that since the data at nearby nodes are likely to be *correlated*, we could try and achieve some sort of information-theoretic compression while transmitting the data from the sensors, as compared to separately outputting the data at each sensor. See [2, 7] for two papers in this growing area; interesting further connections to fields such as information theory and machine learning appear ripe for investigation here.

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