Optimizing Epidemic Protection for Socially Essential Workers

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Epidemic Response

- Public health authorities deploy interventions to contain an epidemic.
  - Vaccination, anti-virals, school closures, etc.
- Trying to minimize the negative effects:
  - Number of deaths, infections, lost work days.
- There is much work on these problems.
  - However protecting the *general* population is for another talk.
Today’s Topic

- Protecting a *societally critical subpopulation*.
  - Doctors, utility workers, military, etc.
  - If large numbers are incapacitated, essential services may stop.
- Overlooked because it *indirectly* benefits society.
  - Hard to quantify the benefit vs. fewer total infections.
Sequestering

• Protect people by isolating them in groups.
  • Military barracks/ship, within a power plant, etc.
  • High connectivity within a group.
  • Zero connectivity between groups.
  • Zero connectivity to the outside.
• Despite best efforts, some will bring the infection.
Sequestering (cont)

- Sequestering starts at some trigger event.
- Critical workers are removed from the population.
- We estimate latent infection probabilities.
- Place them to minimize new infections.
- Our results:
  - How to assign people to groups? We have an efficient, optimal solution.
  - How can we balance the trade-offs between timing, group size, and total infections.
In Group Epidemic Model

- Uniform mixing within a group.
  - Pairwise transmissions with probability $p$.
  - Depends on disease and logistical parameters (sanitation, proximity, speed of diagnosis, etc).
  - Equivalent to finding connected components on a random subgraph $G(n, p)$ where a single infected member infects the entire component.

- Known expected probabilities of infection (EPIs).
  - Otherwise people are interchangeable.
Group Placement Problem

- **Input:**
  - A list of external probabilities of infection (EPI).
  - A list of group capacities.
  - Person-to-person transmission probability $p$.

- **Output:**
  - Assignment to groups that minimizes total expected infections.
  - The expected number of total infections.
Algorithm Overview

• Sort people by their infection probabilities.
• Cut the list into contiguous segments, each one is a group.
• The correctness depends on two things:
  • A proof that optimal partitions are grouped this way.
  • Finding optimal places to cut.
“Well-Ordered” Property

**Theorem 1.** If \( n \) people with external infection probabilities \( s_1 \leq \ldots \leq s_n \) are to be divided into two groups sized \( a \) and \( b \), an optimal solution either places \( 1, \ldots, a \) or \( 1, \ldots, b \) alone in one group, and the remainder in the other group.

**Corollary 2.** Each group in an optimal solution consists of a single range of people from the sorted \( s_i \) list.

- Theorem 1 totally orders the groups, so we can sort them.
Example Group Assignments

- Two sample assignments follow, both show the same random edge connectivities.
- Mixed groups produce more total infections.
A, B, and C are interchangeable pieces.

Expected infections is a function of a person’s EPI and their environment.
Difference Within a Group

- Within a group, people with higher EPIs have more sensitive environments.
- The environment has fewer infections independently from the selected person.
Difference Between Groups

• If a partition is not well-ordered, consider a swap.
• If the higher EPI has a more sensitive environment, the cost goes down.
• At least one swap fits this pattern.
• So the not well-ordered grouping is not optimal.
Finding an Optimal Solution

- Corollary 2 says an optimal solution sorts everyone by $s_i$ and partitions adjacent people into groups.
- How full should each group be?
- With different group capacities, which goes first?
  - Lowest $s_i$’s in the smallest group, largest group, or somewhere in between?
- Figure here showing cutting the sorted list
Expected Group Cost

• Given people with values $s_1, \ldots, s_l$ in a single group, what is the expected cost?
• This can be done exactly using dynamic programming.
• Alternatively this can be done with arbitrary precision through repeated sampling.
  • Does not require the same symmetry.
Placement Algorithm Sketch

- Assumes all groups have the same constraints (otherwise exponential in the number of group types).
- Use dynamic programming:
  - $B[i, j] =$ the minimum # infections using people 1..i and groups 1..j.
  - $B[i, j] = \min_k E[X_{(i-k+1)\ldots i}] + B[i - k, j - 1]$.
- Backtrack through the array to find which assignments led to the minimum expected infections.
Group Size Heuristics

- Smaller groups mean smaller connected components.
  - Each latently infected node leads to fewer final infections.
  - Less likely the group has an infection.
  - If size times $p < 1$, all components are small.
- How much benefit does a smaller group provide?
Outbreaks in Groups

- Talk about Erdos/Renyi random graphs and how sometimes there’s a giant component, sometimes not.
Group Size and Infections

- What are the tradeoffs between group size, latent infection rate, and final infections?
- Assuming uniform latent infections:

\[ p = 0.05 \]

\[ p = 0.1 \]
Effects of Sequestering Timing

• We examine # of infections vs. sequestering timing.

\[ p = 0.05 \] 

\[ p = 0.1 \]
Sequestering Trigger

- In a real-world scenario, we need a measurable trigger.
  - Reported cases among critical population.
  - Reporting lags behind incidences.
- Simulations suggest that to be effective, the threshold must be set very low - less than 1%.
  - Based on a worst case disease.
  - False positives become an issue.
Outside Today’s Scope

• When to return to the general population.
• Improvement of optimal sequestering vs. random.
• Sensitivity to latent probability estimation error.
Summary

• We create a model for a real-world problem.
• We discover a property of our model, and explore what assumptions can be relaxed.
• Using this property, we develop and implement an efficient algorithm.
• Using our algorithm, we explore other design tradeoffs.
• Our results can help public health decision makers.