Two-Factor ANOVA

two independent variables

example: independent measures, equal n designs

	JVM Heap size: 128 MB	256 MB	512 MB
sun java	n = 15 java programs	n = 15	n = 15
jikes	n = 15	n = 15	n = 15

	JVM Heap Size: 128 MB	256 MB	512 MB
sun java	$n = 15$ java programs	$n = 15$	$n = 15$
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3 Hypothesis Tests

① $H_0: \mu_{java} = \mu_{jikes}$

② $H_0: \mu_{128} = \mu_{256} = \mu_{512}$

③ H_0: no interaction b/w heap size, compiler

3 Hypothesis Tests

① $H_0: \mu_{A1} = \mu_{A2}$

② $H_0: \mu_{B1} = \mu_{B2} = \mu_{B3}$

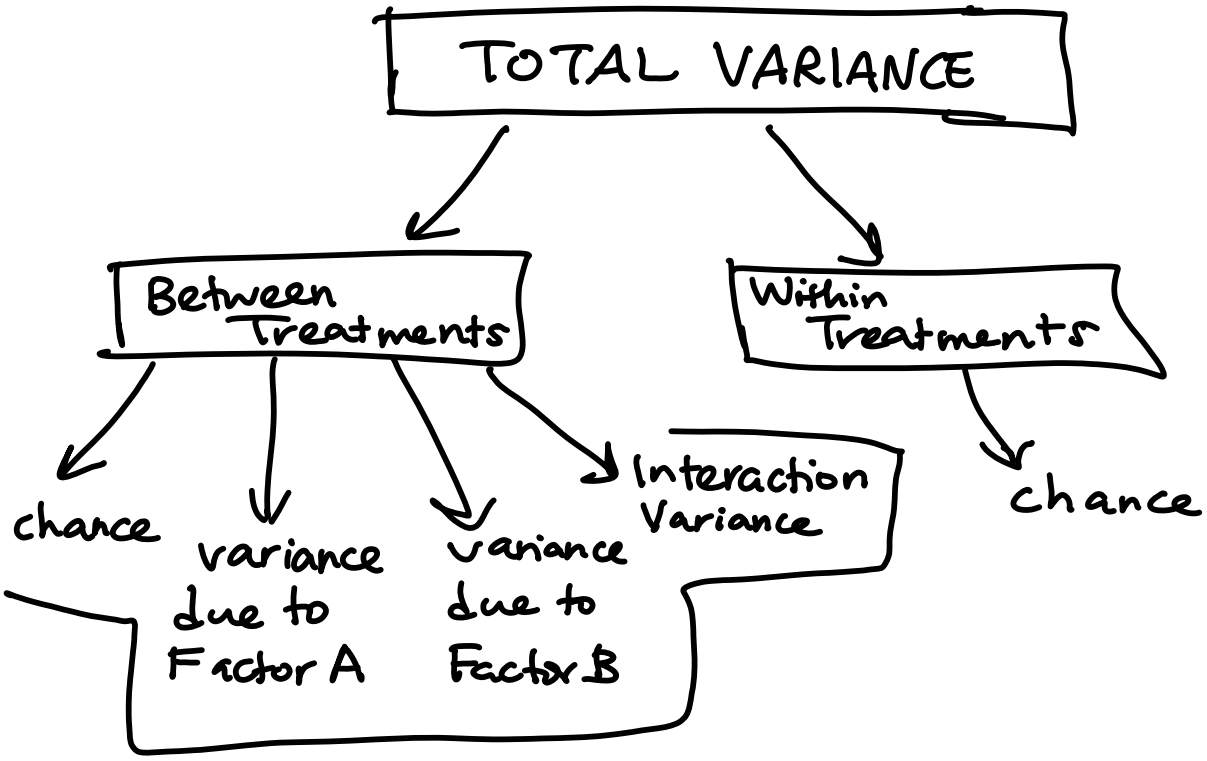
} main effects

③ H_0: no interaction b/w A and B

(A×B interaction: effect of A depends on level of B and/or vice versa)

For each test, compute corresponding F-statistic.

Main Effects: $F_A = \dfrac{\text{variance between means for factor A}}{\text{variance expected by chance}}$


TOTAL VARIANCE
Between Treatments
Within Treatments
chance
variance due to Factor A
variance due to Factor B
Interaction Variance
chance

	B_1	B_2	B_3	ROW
A_1	3, 1, 1, 6, 4 $\bar{X}=3$, $SS=18$	2, 5, 9, 7, 7 $\bar{X}=6$, $SS=28$	9, 9, 13, 6, 8 $\bar{X}=9$, $SS=26$	$n=15$ TOTAL = 90 MEAN = 6
A_2	0, 2, 0, 0, 3 $\bar{X}=1$, $SS=8$	3, 8, 3, 3, 3 $\bar{X}=4$, $SS=20$	0, 0, 0, 5, 0 $\bar{X}=1$, $SS=20$	$n=15$ TOTAL = 30 MEAN = 2
COL	$n=10$ TOTAL = 20 MEAN = 2	$n=10$ TOTAL = 50 MEAN = 5	$n=10$ TOTAL = 50 MEAN = 5	

$N = 5 \times 6 = 30$

TOTAL = 120

OVERALL MEAN = 4

Source	SS	df	var	F
b/w treatments	240	5		
Factor A	120	1	120	120/5 = 24
Factor B	60	2	30	30/5 = 6
A x B	60	2	30	30/5 = 6
Within Treatments	120	24	120/24 = 5	
Total.	360	29		

$\alpha = 0.05$ $F(1, 24) = 4.26$ $24 >> 4.26$

$F(2, 24) = 3.40$ $6 > 3.40$

df

$N - 1 = 29.$

$6 - 1 = 5$

$2 - 1 = 1$

$3 - 1 = 2$

$A \times B = 5 - 1 - 2 = 2$

Within = Total − b/w = $29 - 5 = 24$

SS

overall mean

TOTAL: $(3-4)^2 + (1-4)^2 + (1-4)^2$

$+ \ldots + (5-4)^2 + (0-4)^2 = 360$

SS within treatments

$= 18 + 28 + 26 + 8 + 20 + 20$

$= 120$

				ROW
A_1	3, 1, 1, 6, 4	2, 5, 9, 7, 7	9, 9, 3, 6, 8	n = 15 TOTAL = 90 MEAN = 6
A_2	0, 2, 0, 0, 3	3, 8, 3, 3, 3	0, 0, 0, 5, 0	n = 15 TOTAL = 30 MEAN = 2

OVERALL MEAN = 4

$$SS_A = n_A \left[\sum_{i=1}^{2} \left(\bar{X}_{Ai} - \text{OVERALL MEAN} \right)^2 \right]$$

$$= 15 \left(2^2 + 2^2 \right) = 120$$

B_1	B_2	B_3
3, 1, 1, 6, 4 0, 2, 0, 0, 3	2, 5, 9, 7, 7 3, 8, 3, 3, 3	9, 9, 13, 6, 8 0, 0, 0, 5, 0
$n = 10$ TOTAL = 20 MEAN = 2	$n = 10$ TOTAL = 50 MEAN = 5	$n = 10$ TOTAL = 50 MEAN = 5

COL

OVERALL MEAN = 4

$$SS_B = n_B \left(\sum \left(\bar{x}_{B_i} - \text{OVERALL MEAN} \right)^2 \right)$$

$$= 10 \cdot (2^2 + 1 + 1) = 60$$

Assumptions of Two-Factor ANOVA (indep. measures)

1. observations within each sample are indep.
2. popns are normal
3. popns have homogeneity of variance