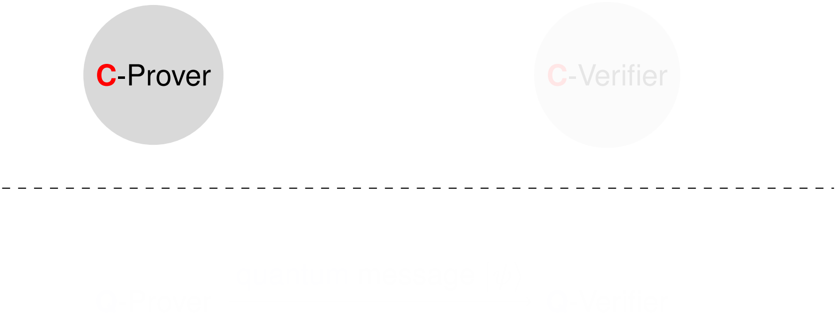
Epsilon-net method for optimizations over separable states

Yaoyun Shi and Xiaodi Wu

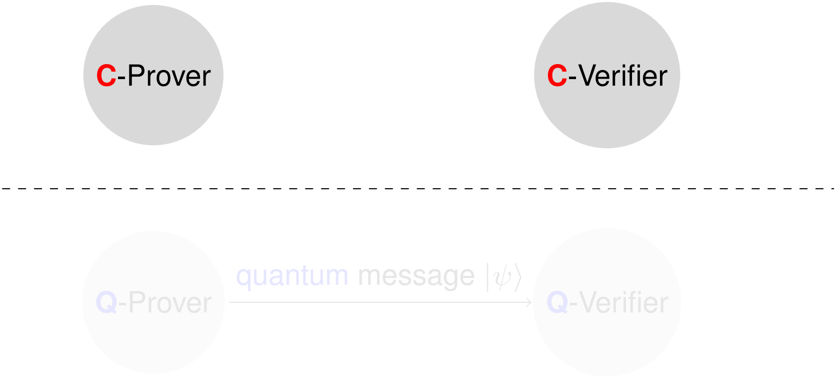
Department of Electrical Engineering and Computer Science
University of Michigan, Ann Arbor

ICALP 2012, July 2012

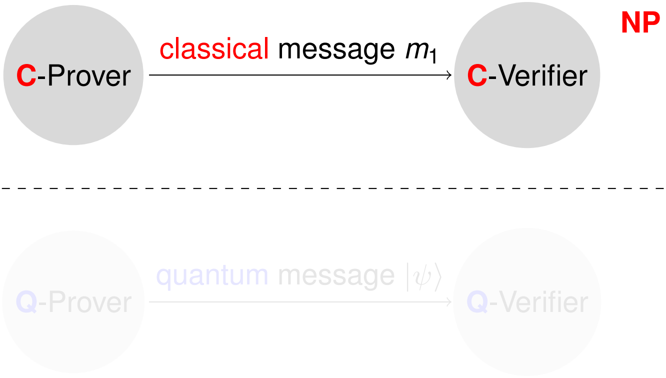
Main Motivation: QMA(2) vs QMA



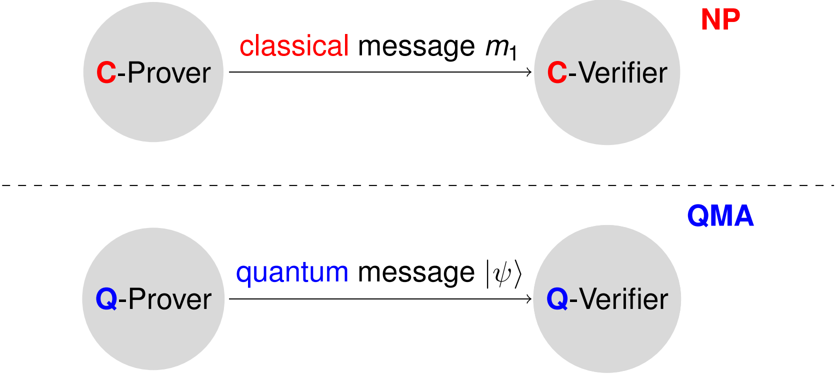
Main Motivation: QMA(2) vs QMA



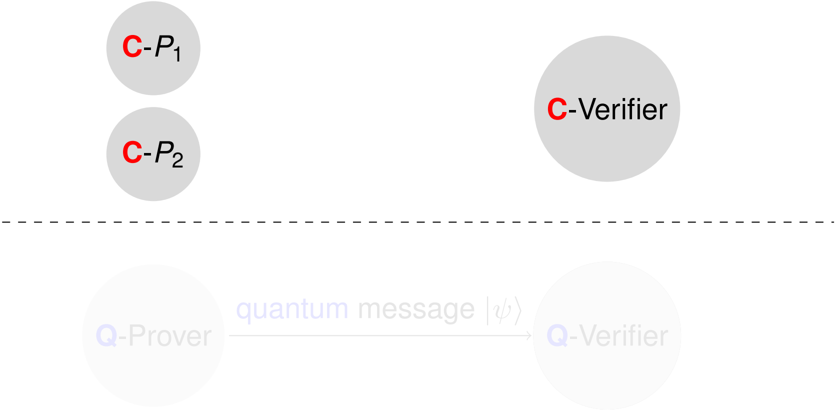
Main Motivation: QMA(2) vs QMA



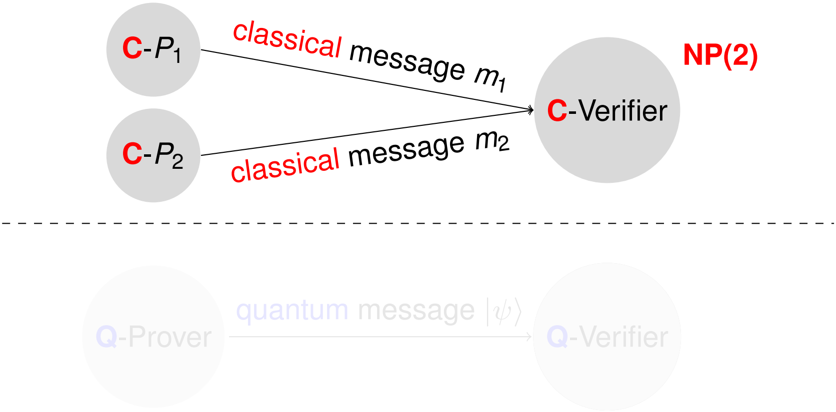
Main Motivation: QMA(2) vs QMA



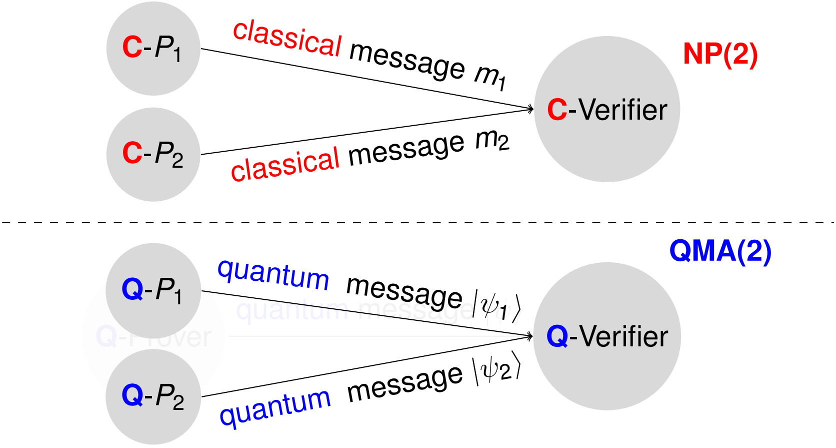
Main Motivation: QMA(2) vs QMA



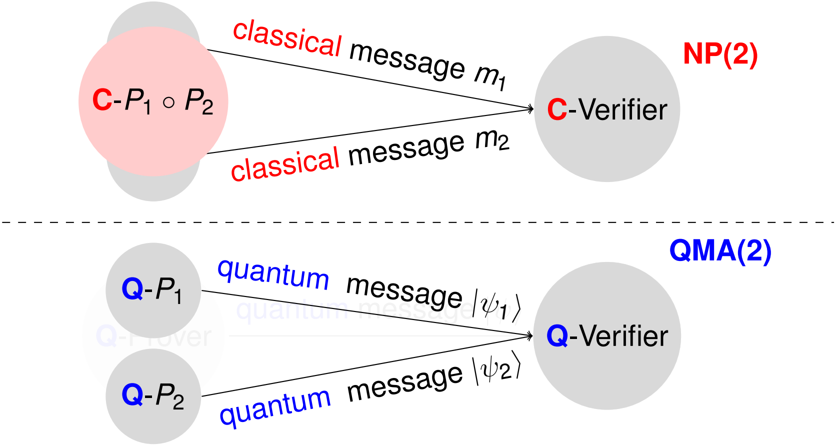
Main Motivation: QMA(2) vs QMA



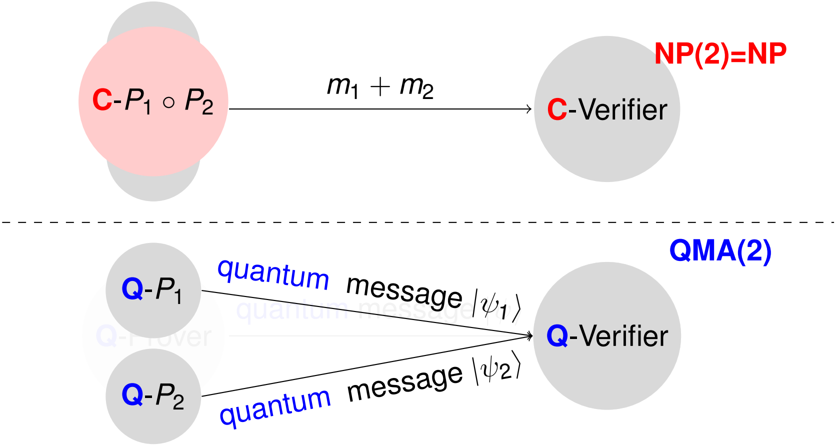
Main Motivation: QMA(2) vs QMA



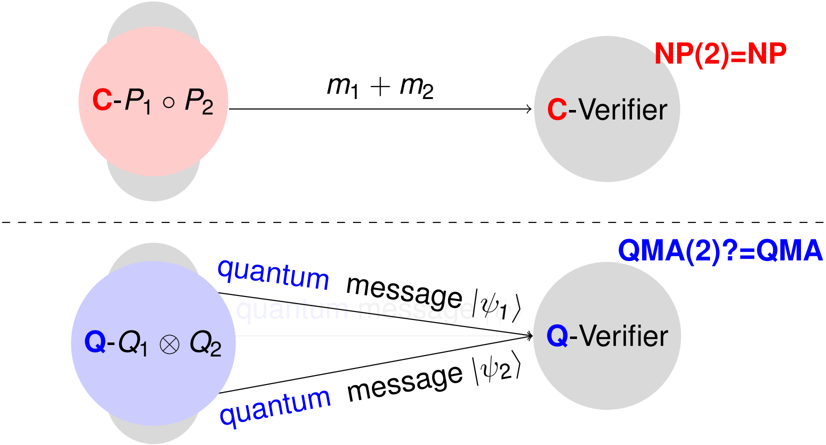
Main Motivation: QMA(2) vs QMA



Main Motivation: QMA(2) vs QMA



Main Motivation: QMA(2) vs QMA



History about QMA(2)

- Introduced in [KMY01, KMY03].
- Surprising: NP $\subseteq$ QMA$(2)_{\log}$ [BT09] comparing with QMA$_{\log}$ $=$ BQP [MW05]. Trivially, NP$_{\log}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2) , e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

History about QMA(2)

- Introduced in [KMY01, KMY03].
- **Surprising**: NP $\subseteq$ QMA(2)$_{\log}$ [BT09] comparing with QMA$_{\log}$ = BQP [MW05]. Trivially, NP$_{\log}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2), e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

History about QMA(2)

- Introduced in [KMY01, KMY03].
- **Surprising**: NP $\subseteq$ QMA(2)$_{\log}$ [BT09] comparing with QMA$_{\log}$ = BQP [MW05]. Trivially, NP$_{\log}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2), e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

History about QMA(2)

- Introduced in [KMY01, KMY03].
- **Surprising**: NP $\subseteq$ QMA(2)$_{\text{log}}$ [BT09] comparing with QMA$_{\text{log}}$ $=$ BQP [MW05]. Trivially, NP$_{\text{log}}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2) , e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

History about QMA(2)

- Introduced in [KMY01, KMY03].
- **Surprising**: NP $\subseteq$ QMA(2)$_{\log}$ [BT09] comparing with QMA$_{\log}$ $=$ BQP [MW05]. Trivially, NP$_{\log}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2) , e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

History about QMA(2)

- Introduced in [KMY01, KMY03].
- **Surprising**: NP $\subseteq$ QMA(2)$_{\log}$ [BT09] comparing with QMA$_{\log}$ $=$ BQP [MW05]. Trivially, NP$_{\log}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2) , e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

History about QMA(2)

- Introduced in [KMY01, KMY03].
- **Surprising**: NP $\subseteq$ QMA(2)$_{\log}$ [BT09] comparing with QMA$_{\log}$ $=$ BQP [MW05]. Trivially, NP$_{\log}$ $\subseteq$ P.
- Various improvements [Bei10, ABD+09, CD10, CF11, GNN11, ...].

Trivially, QMA(2)$\subseteq$NEXP.

- Variants of QMA(2) , e.g. BellQMA, LOCC-QMA, collapse to QMA [Bra08, ABD+09, BCY11].
- QMA(2)=QMA(poly) [HM10].

Better upper bounds, such as EXP, PSPACE, are expected.

Quantum Notations

- *Density Operators:* Representation of quantum states.
  Note: n-qubit quantum state requires 2^n by 2^n matrix.
- *Measurements:* The outcome (e.g., *probability*) of a quantum circuit is given by the *inner product* $\langle M, \rho \rangle$ where M is a PSD defined by the circuit.
- *Tensor Product:* For any *isolated* two systems, the quantum state of the whole state is $\rho \otimes \sigma$ where ρ is the density operator from the first system while σ is from the other one.

Quantum Notations

- *Density Operators:* Representation of quantum states.
  Note: n-qubit quantum state requires 2^n by 2^n matrix.
- *Measurements:* The outcome (e.g., *probability*) of a quantum circuit is given by the *inner product* $\langle M, \rho \rangle$ where M is a PSD defined by the circuit.
- *Tensor Product:* For any *isolated* two systems, the quantum state of the whole state is $\rho \otimes \sigma$ where ρ is the density operator from the first system while σ is from the other one.

Quantum Notations

- *Density Operators:* Representation of quantum states.
  Note: n-qubit quantum state requires 2^n by 2^n matrix.
- *Measurements:* The outcome (e.g., *probability*) of a quantum circuit is given by the *inner product* $\langle M, \rho \rangle$ where M is a PSD defined by the circuit.
- *Tensor Product:* For any *isolated* two systems, the quantum state of the whole state is $\rho \otimes \sigma$ where ρ is the density operator from the first system while σ is from the other one.

Formulation of QMA(2)

Definition (QMA(2))

A language $\mathcal{L}$ is in QMA(2) if there exists a polynomial-time generated two-outcome measurement $\{Q_x^{\text{acc}}, I - Q_x^{\text{acc}}\}$ s.t.,

- If $x \in \mathcal{L}, \exists \rho_1, \rho_2, \langle Q_x^{\text{acc}}, \rho_1 \otimes \rho_2 \rangle \geq \frac{2}{3}$.
- If $x \notin \mathcal{L}, \forall \rho_1, \rho_2, \langle Q_x^{\text{acc}}, \rho_1 \otimes \rho_2 \rangle \leq \frac{1}{3}$.

Roughly equivalent to computing $\max \langle \mathbf{Q}_\mathbf{x}^{\text{acc}}, \rho_1 \otimes \rho_2 \rangle$, except

- Larger additive error allowed.
- Special and possibly nicer Q_x^{acc}'s by *manipulating the protocol*.

Formulation of QMA(2)

Definition (QMA(2))

A language $\mathcal{L}$ is in QMA(2) if there exists a polynomial-time generated two-outcome measurement $\{Q_x^{acc}, I - Q_x^{acc}\}$ s.t.,

- If $x \in \mathcal{L}, \exists \rho_1, \rho_2, \langle Q_x^{acc}, \rho_1 \otimes \rho_2 \rangle \geq \frac{2}{3}$.
- If $x \notin \mathcal{L}, \forall \rho_1, \rho_2, \langle Q_x^{acc}, \rho_1 \otimes \rho_2 \rangle \leq \frac{1}{3}$.

Roughly equivalent to computing $\max \langle \mathbf{Q_x^{acc}}, \rho_\mathbf{1} \otimes \rho_\mathbf{2} \rangle$, except

- Larger additive error allowed.
- Special and possibly nicer Q_x^{acc}s by *manipulating the protocol*.

Formulation of QMA(2)

Definition (QMA(2))

A language $\mathcal{L}$ is in QMA(2) if there exists a polynomial-time generated two-outcome measurement $\{Q_x^{\text{acc}}, I - Q_x^{\text{acc}}\}$ s.t.,

- If $x \in \mathcal{L}, \exists \rho_1, \rho_2, \langle Q_x^{\text{acc}}, \rho_1 \otimes \rho_2 \rangle \geq \frac{2}{3}$.
- If $x \notin \mathcal{L}, \forall \rho_1, \rho_2, \langle Q_x^{\text{acc}}, \rho_1 \otimes \rho_2 \rangle \leq \frac{1}{3}$.

Roughly equivalent to computing $\max \langle \mathbf{Q_x^{\text{acc}}}, \rho_1 \otimes \rho_2 \rangle$, except

- Larger additive error allowed.
- Special and possibly nicer Q_x^{acc}s by *manipulating the protocol*.

Formulation of QMA(2)

Definition (QMA(2))

A language $\mathcal{L}$ is in QMA(2) if there exists a polynomial-time generated two-outcome measurement $\{Q_x^{acc}, I - Q_x^{acc}\}$ s.t.,

- If $x \in \mathcal{L}, \exists \rho_1, \rho_2, \langle Q_x^{acc}, \rho_1 \otimes \rho_2 \rangle \geq \frac{2}{3}$.
- If $x \notin \mathcal{L}, \forall \rho_1, \rho_2, \langle Q_x^{acc}, \rho_1 \otimes \rho_2 \rangle \leq \frac{1}{3}$.

Roughly equivalent to computing $\max \langle \mathbf{Q_x^{acc}}, \rho_1 \otimes \rho_2 \rangle$, except

- Larger additive error allowed.
- Special and possibly nicer Q_x^{acc}s by *manipulating the protocol*.

The Problem

Problem (Quantum Formulation)

Given $\mathbf{H} \in \mathrm{Herm}(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \textbf{subject to}\ \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y}),$$

where $\mathrm{D}(\mathcal{X})$ *is the set of trace-one psd matrices over* $\mathcal{X}$.

- Ground energy that is achieved by *non-entangled* states.
- *Mean-field* approximation in statistical quantum mechanics.
- *Positivity* test of quantum channels.
- *17 more examples* in quantum information in [HM10].

The Problem

Problem (Quantum Formulation)

Given $\mathbf{H} \in \mathrm{Herm}(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \textbf{subject to } \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y}),$$

where $\mathrm{D}(\mathcal{X})$ *is the set of trace-one psd matrices over* $\mathcal{X}$.

- Ground energy that is achieved by *non-entangled* states.
- *Mean-field* approximation in statistical quantum mechanics.
- *Positivity* test of quantum channels.
- *17 more examples* in quantum information in [HM10].

The Problem

Problem (Quantum Formulation)

Given $\mathbf{H} \in \mathrm{Herm}(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \textbf{subject to } \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y}),$$

where $\mathrm{D}(\mathcal{X})$ *is the set of trace-one psd matrices over* $\mathcal{X}$.

- Ground energy that is achieved by *non-entangled* states.
- *Mean-field* approximation in statistical quantum mechanics.
- *Positivity* test of quantum channels.
- *17 more examples* in quantum information in [HM10].

The Problem

Problem (Quantum Formulation)

Given $\mathbf{H} \in \mathrm{Herm}(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \textbf{subject to} \ \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y}),$$

where $\mathrm{D}(\mathcal{X})$ *is the set of trace-one psd matrices over* $\mathcal{X}$.

- Ground energy that is achieved by *non-entangled* states.
- *Mean-field* approximation in statistical quantum mechanics.
- *Positivity* test of quantum channels.
- *17 more examples* in quantum information in [HM10].

The Problem

Problem (Quantum Formulation)

Given $\mathbf{H} \in \mathrm{Herm}(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \textbf{subject to}\ \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y}),$$

where $\mathrm{D}(\mathcal{X})$ *is the set of trace-one psd matrices over* $\mathcal{X}$.

- Ground energy that is achieved by *non-entangled* states.
- *Mean-field* approximation in statistical quantum mechanics.
- *Positivity* test of quantum channels.
- *17 more examples* in quantum information in [HM10].

The Problem : Classical Formulation

Problem (Quantum Formulation)

Given $\mathbf{H} \in \mathrm{Herm}(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \textbf{subject to} \ \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y}),$$

where $\mathrm{D}(\mathcal{X})$ *is the set of trace-one psd matrices over* $\mathcal{X}$*.*

is roughly equivalent to

Problem (Classical Formulation)

Given $\mathbf{H} \in Sym(\mathcal{X} \otimes \mathcal{Y})$ *as input, compute*

$$\max \sum_{i,j,k,l} H_{ij,kl} x_i y_j x_k y_l \ \textit{subject to} \ \sum_i x_i^2 = \sum_i y_i^2 = 1.$$

A special class of the polynomial optimization problems.

More Motivations

- **Quantum Information** : more examples in [HM10].
- **Quantum Computational Complexity**: QMA(2).
- **Operations Research**: "Bi-Quadratic Optimization over Unit Spheres" [LNQY09]. Polynomial Optimization with Quadratic Constraints.
- **Unique Game Conjecture**: 2-to-4 norm, Small-Set Expansion-hardness [BBHKSZ12].

More Motivations

- **Quantum Information** : more examples in [HM10].
- **Quantum Computational Complexity**: QMA(2).
- **Operations Research**: "Bi-Quadratic Optimization over Unit Spheres" [LNQY09]. Polynomial Optimization with Quadratic Constraints.
- **Unique Game Conjecture**: 2-to-4 norm, Small-Set Expansion-hardness [BBHKSZ12].

More Motivations

- **Quantum Information** : more examples in [HM10].
- **Quantum Computational Complexity**: QMA(2).
- **Operations Research**: "Bi-Quadratic Optimization over Unit Spheres" [LNQY09]. Polynomial Optimization with Quadratic Constraints.
- **Unique Game Conjecture**: 2-to-4 norm, Small-Set Expansion-hardness [BBHKSZ12].

More Motivations

- **Quantum Information** : more examples in [HM10].
- **Quantum Computational Complexity**: QMA(2).
- **Operations Research**: “Bi-Quadratic Optimization over Unit Spheres” [LNQY09]. Polynomial Optimization with Quadratic Constraints.
- **Unique Game Conjecture**: 2-to-4 norm, Small-Set Expansion-hardness [BBHKSZ12].

The Problem: Easiness vs Hardness

EASY:

$$\max \langle \mathbf{H}, \rho \rangle \text{ s.t. } \rho \in \mathrm{D}(\mathcal{X} \otimes \mathcal{Y})$$

- Efficiently solvable via the *spectral decomposition* of H.

HARD:

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \text{ s.t. } \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y})$$

- NP-hard even to approximate the optimum value with *inverse-polynomial* additive error.
- Hardness via quantum information [Gur03,Ioa07,Gha10] or operation research [deK08, LQNY09].

The Problem: Easiness vs Hardness

EASY:

$$\max \langle \mathbf{H}, \rho \rangle \text{ s.t. } \rho \in \mathrm{D}(\mathcal{X} \otimes \mathcal{Y})$$

- Efficiently solvable via the *spectral decomposition* of H.

HARD:

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \text{ s.t. } \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y})$$

- **NP-hard** even to approximate the optimum value with *inverse-polynomial* additive error.
- Hardness via quantum information [Gur03,Ioa07,Gha10] or operation research [deK08, LQNY09].

The Problem: Easiness vs Hardness

EASY:

$$\max \langle \mathbf{H}, \rho \rangle \ \text{s.t.}\ \rho \in \mathrm{D}(\mathcal{X} \otimes \mathcal{Y})$$

- Efficiently solvable via the *spectral decomposition* of H.

HARD:

$$\max \langle \mathbf{H}, \rho \otimes \sigma \rangle \ \text{s.t.}\ \rho \in \mathrm{D}(\mathcal{X}), \sigma \in \mathrm{D}(\mathcal{Y})$$

- **NP-hard** even to approximate the optimum value with *inverse-polynomial* additive error.
- Hardness via quantum information [Gur03,Ioa07,Gha10] or operation research [deK08, LQNY09].

Our Results

RESULT 1: making use of the **DECOMPOSABILITY** of **H**.

- **Time** and **Space** -efficient algorithms when $\mathbf{H} = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ with small M.
- Applied in *quantum computational complexity*, we prove QMA(2)[poly(n),$O(\log(n))$]$\subseteq$ PSPACE

RESULT 2: making use of the **EIGENSPACE** of **H**.

- Time complexity $\exp(O(\log(d) + \delta^{-2}\|\mathbf{H}\|_F^2 \ln(\|\mathbf{H}\|_F/\delta)))$ with additive error δ for $\mathbf{H} \geq 0$.
- Conceptually simpler and better running time than an earlier algorithm [BCY11] (time complexity $\exp(O(\log^2(d)\delta^{-2}\|\mathbf{H}\|_F^2))$, using *symmetric extension*, *quantum de Finetti bounds*).

Our Results

RESULT 1: making use of the **DECOMPOSABILITY** of **H**.

- **Time** and **Space** -efficient algorithms when $\mathbf{H} = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ with small M.
- Applied in *quantum computational complexity*, we prove QMA(2)[poly(n),O(log(n))]$\subseteq$ PSPACE

RESULT 2: making use of the **EIGENSPACE** of **H**.

- Time complexity $\exp(O(\log(d) + \delta^{-2}\|\mathbf{H}\|_F^2 \ln(\|\mathbf{H}\|_F/\delta)))$ with additive error δ for $\mathbf{H} \geq 0$.
- Conceptually simpler and better running time than an earlier algorithm [BCY11] (time complexity $\exp(O(\log^2(d)\delta^{-2}\|\mathbf{H}\|_F^2))$, using *symmetric extension, quantum de Finetti bounds*).

Our Results

RESULT 1: making use of the **DECOMPOSABILITY** of **H**.

- **Time** and **Space** -efficient algorithms when $\mathbf{H} = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ with small M.
- Applied in *quantum computational complexity*, we prove QMA(2)[poly(n),$O(\log(n))$]$\subseteq$ PSPACE

RESULT 2: making use of the **EIGENSPACE** of **H**.

- Time complexity $\exp(O(\log(d) + \delta^{-2}\|\mathbf{H}\|_F^2 \ln(\|\mathbf{H}\|_F/\delta)))$ with additive error δ for $\mathbf{H} \geq 0$.
- Conceptually simpler and better running time than an earlier algorithm [BCY11] (time complexity $\exp(O(\log^2(d)\delta^{-2}\|\mathbf{H}\|_F^2))$, using *symmetric extension, quantum de Finetti bounds*).

Our Results

RESULT 1: making use of the **DECOMPOSABILITY** of **H**.

- **Time** and **Space** -efficient algorithms when $\mathbf{H} = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ with small M.
- Applied in *quantum computational complexity*, we prove QMA(2)[poly(n),$O(\log(n))$]$\subseteq$ PSPACE

RESULT 2: making use of the **EIGENSPACE** of **H**.

- Time complexity $\exp(O(\log(d) + \delta^{-2}\|\mathbf{H}\|_F^2 \ln(\|\mathbf{H}\|_F/\delta)))$ with additive error δ for $\mathbf{H} \geq 0$.
- Conceptually simpler and better running time than an earlier algorithm [BCY11] (time complexity $\exp(O(\log^2(d)\delta^{-2}\|\mathbf{H}\|_F^2))$, using *symmetric extension, quantum de Finetti bounds*).

Our Results

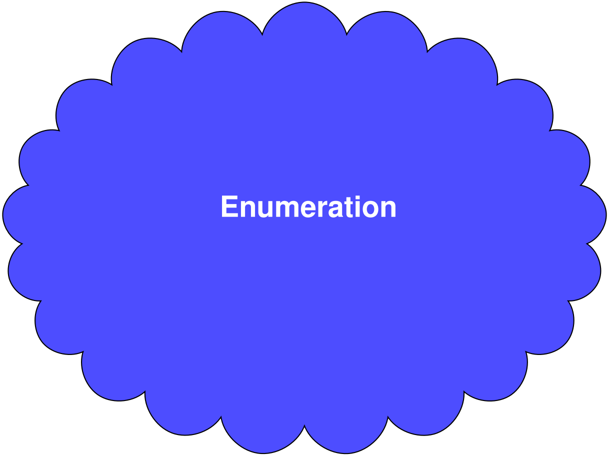
RESULT 1: making use of the **DECOMPOSABILITY** of **H**.

- **Time** and **Space** -efficient algorithms when $\mathbf{H} = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ with small M.
- Applied in *quantum computational complexity*, we prove QMA(2)[poly(n),$O(\log(n))$]$\subseteq$ PSPACE

RESULT 2: making use of the **EIGENSPACE** of **H**. Omitted!

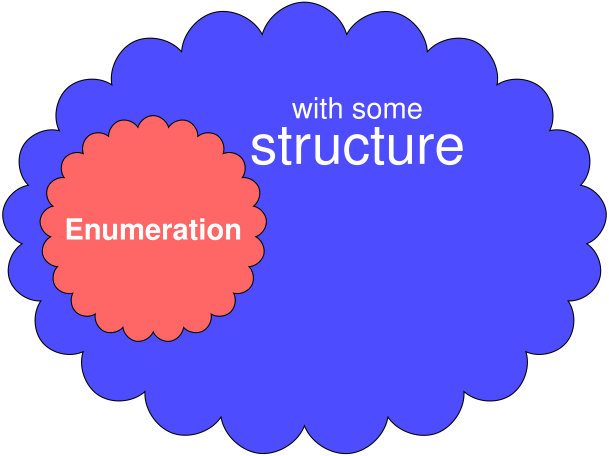
- Time complexity $\exp(O(\log(d) + \delta^{-2}\|\mathbf{H}\|_F^2 \ln(\|\mathbf{H}\|_F/\delta)))$ with additive error δ for $\mathbf{H} \geq 0$.
- Conceptually simpler and better running time than an earlier algorithm [BCY11] (time complexity $\exp(O(\log^2(d)\delta^{-2}\|\mathbf{H}\|_F^2))$, using *symmetric extension, quantum de Finetti bounds*).

High-level Technique Overview

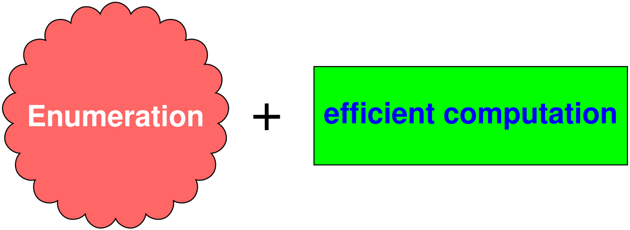


High-level Technique Overview

High-level Technique Overview



High-level Technique Overview



Result based on the DECOMPOSABILITY of H

Definition

We call **H** is *$(M, \vec{w})$-decomposable* if $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ where $\| H_i^1 \| \leq w_1, \| H_i^2 \| \leq w_2$.

Intuition: the smaller $M \Rightarrow$ the more "local" **H** and the less connection between the two parties.

- Enumerate and then fix the connection, and solve the optimization separably.
- Assume the decomposition is given or easily computable. *Not necessarily the smallest M.*

We obtain efficient algorithms in both **TIME** and **SPACE** when M is small.

Result based on the DECOMPOSABILITY of H

Definition

We call **H** is *$(M, \vec{w})$-decomposable* if $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ where $\| H_i^1 \| \leq w_1, \| H_i^2 \| \leq w_2$.

Intuition: the smaller $M \Rightarrow$ the more "local" **H** and the less connection between the two parties.

- Enumerate and then fix the connection, and solve the optimization separably.
- Assume the decomposition is given or easily computable. *Not necessarily the smallest M*.

We obtain efficient algorithms in both **TIME** and **SPACE** when M is small.

Result based on the DECOMPOSABILITY of H

Definition

We call **H** is *$(M, \vec{w})$-decomposable* if $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ where $\|H_i^1\| \leq w_1, \|H_i^2\| \leq w_2$.

Intuition: the smaller $M \Rightarrow$ the more "local" **H** and the less connection between the two parties.

- Enumerate and then fix the connection, and solve the optimization separably.
- Assume the decomposition is given or easily computable. *Not necessarily the smallest M*.

We obtain efficient algorithms in both **TIME** and **SPACE** when M is small.

Result based on the DECOMPOSABILITY of H

Definition

We call **H** is *$(M, \vec{w})$-decomposable* if $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$ where $\|H_i^1\| \leq w_1, \|H_i^2\| \leq w_2$.

Intuition: the smaller $M \Rightarrow$ the more "local" **H** and the less connection between the two parties.

- Enumerate and then fix the connection, and solve the optimization separably.
- Assume the decomposition is given or easily computable. *Not necessarily the smallest M*.

We obtain efficient algorithms in both **TIME** and **SPACE** when M is small.

Result based on the DECOMPOSABILITY of H

As a result, we prove **QMA(2)[poly(n),O(log(n))]⊆ PSPACE**.

- Restricted verifier that only performs *poly(n)* type-I elementary gates and $O(\log(n))$ type-II elementary gates.

Type-I: *local gates*

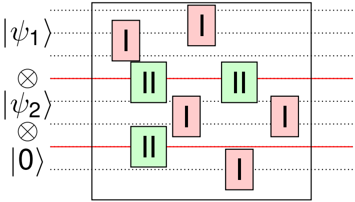
Type-II: *crossing gates*

- Stronger verifier than those in **BellQMA** and **LOCC-QMA**.
- **PSPACE** upper bound.

Result based on the DECOMPOSABILITY of H

As a result, we prove **QMA(2)[poly(n),O(log(n))]$\subseteq$ PSPACE**.

- Restricted verifier that only performs *poly(n)* type-I elementary gates and $O(\log(n))$ type-II elementary gates.



Type-I: *local gates*

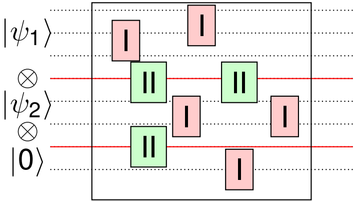
Type-II: *crossing gates*

- Stronger verifier than those in **BellQMA** and **LOCC-QMA**.
- **PSPACE** upper bound.

Result based on the DECOMPOSABILITY of H

As a result, we prove **QMA(2)[poly(n),O(log(n))]$\subseteq$ PSPACE**.

- Restricted verifier that only performs *poly(n)* type-I elementary gates and $O(\log(n))$ type-II elementary gates.



Type-I: *local gates*

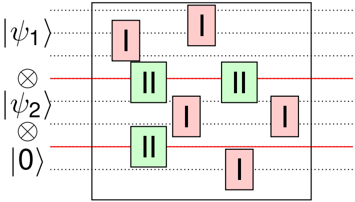
Type-II: *crossing gates*

- Stronger verifier than those in **BellQMA** and **LOCC-QMA**.
- **PSPACE** upper bound.

Result based on the DECOMPOSABILITY of H

As a result, we prove **QMA(2)[poly(n),O(log(n))]$\subseteq$ PSPACE**.

- Restricted verifier that only performs *poly(n)* type-I elementary gates and $O(\log(n))$ type-II elementary gates.



Type-I: *local gates*

Type-II: *crossing gates*

- Stronger verifier than those in **BellQMA** and **LOCC-QMA**.
- **PSPACE** upper bound.

CONNECTION enumeration, TIME efficiency

Assume $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$,

$$\max \langle H, \rho_1 \otimes \rho_2 \rangle = \max \langle H_1^1, \rho_1 \rangle \langle H_1^2, \rho_2 \rangle + \cdots + \langle H_M^1, \rho_1 \rangle \langle \mathcal{H}_M^2, \rho_2 \rangle$$

- **HARD**: because of the product ( ) terms.
- **EASY** (*efficiently solvable*) : when the values are fixed.
- Enumerate the valid values of the terms. *Details later.*
- Small $M \Rightarrow$ enumeration space is small.

CONNECTION enumeration, TIME efficiency

Assume $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$,

$$\max \langle H, \rho_1 \otimes \rho_2 \rangle = \max \langle H_1^1, \rho_1 \rangle \langle H_1^2, \rho_2 \rangle + \cdots + \langle H_M^1, \rho_1 \rangle \langle \mathcal{H}_M^2, \rho_2 \rangle$$

- **HARD**: because of the product (○ ○) terms.
- **EASY** (*efficiently solvable*) : when the ○ values are fixed.
- Enumerate the valid values of the ○ terms. *Details later.*
- Small $M \Rightarrow$ enumeration space is small.

CONNECTION enumeration, TIME efficiency

Assume $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$,

$$\max \langle H, \rho_1 \otimes \rho_2 \rangle = \max \langle H_1^1, \rho_1 \rangle \langle H_1^2, \rho_2 \rangle + \cdots + \langle H_M^1, \rho_1 \rangle \langle \mathcal{H}_M^2, \rho_2 \rangle$$

- **HARD**: because of the product (○ ○) terms.
- **EASY** (*efficiently solvable*) : when the ○ values are fixed.
- Enumerate the valid values of the ○ terms. *Details later.*
- Small $M \Rightarrow$ enumeration space is small.

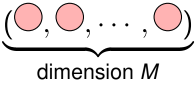
CONNECTION enumeration, TIME efficiency

Assume $H = \sum_{i=1}^{M} H_i^1 \otimes H_i^2$,

$$\max \langle H, \rho_1 \otimes \rho_2 \rangle = \max \langle H_1^1, \rho_1 \rangle \langle H_1^2, \rho_2 \rangle + \cdots + \langle H_M^1, \rho_1 \rangle \langle \mathcal{H}_M^2, \rho_2 \rangle$$

- **HARD**: because of the product (○ ○) terms.
- **EASY** (*efficiently solvable*) : when the ○ values are fixed.
- Enumerate the valid values of the ○ terms. *Details later.*
- Small $M \Rightarrow$ enumeration space is small.

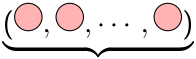
CONNECTION enumeration, SPACE efficiency



from some $\rho_1 \in \mathrm{D}(\mathcal{X})$

- Enumerate raw values of ● terms from a **bounded** set.
- Validness checking by using the **multiplicative weight update method** to compute a **min-max** form.
- **Spectral Decomposition** after the ● values are fixed.

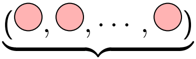
CONNECTION enumeration, SPACE efficiency



from some $\rho_1 \in \mathrm{D}(\mathcal{X})$

- Enumerate raw values of terms from a **bounded** set.
- Validness checking by using the **multiplicative weight update method** to compute a **min-max** form.
- **Spectral Decomposition** after the values are fixed.

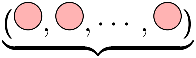
CONNECTION enumeration, SPACE efficiency



from some $\rho_1 \in \mathrm{D}(\mathcal{X})$

- Enumerate raw values of terms from a **bounded** set.
- Validness checking by using the **multiplicative weight update method** to compute a **min**-**max** form.
- **Spectral Decomposition** after the values are fixed.

CONNECTION enumeration, SPACE efficiency

 from some $\rho_1 \in \mathrm{D}(\mathcal{X})$

dimension M

- Enumerate raw values of  terms from a **bounded** set.
  Efficient in both **TIME** and **SPACE**.
- Validness checking by using the **multiplicative weight update method** to compute a **min**-**max** form.
  Efficient in both **TIME** and **SPACE**.
- **Spectral Decomposition** after the  values are fixed.
  Efficient in both **TIME** and **SPACE**.

Summary

In this talk, we provide two algorithms based on the following structures of **H**.

- The decomposability of **H**. PSPACE *upper bound of a new and potentially more powerful QMA(2) variant*.
- The eigenspace of high eigenvalues of **H**.

Open Problems:

- Algorithm or Hardness for larger additive error.
- Upper bound for QMA(2).

Question And Answer

Thank you!