Sketch of Solutions: Homework #1

Note: No partial credits are given to each graded unit.

Problem 1.1.1 [3+3+4=10 points]
(a) true. Every set is a subset of itself, including $\emptyset$. [3 points]
(e) true. {a,b} is a element of {a,b,c,{a,b}}. [3 points]
(g) false. Each element of a subset of a power set is a set, while a, b are not. [4 points]

Problem 1.1.2 [3+3+4=10 points]
(b) {3,5,7} [3 points]
(c) {1,2,7,9} [3 points]
(e) {$\emptyset$}, or { { } } equivalently [4 points]

Problem 1.1.3 [5 points]
(d) If $a \in A$, then $A \in A \cup (A \cap B)$. If $a \notin A$, then $a \notin A \cap B$, thus $a \notin A \cup (A \cap B)$. Finally $A = A \cup (A \cap B)$.

Problem 1.1.4 [5+5=10 points]
(a) Fewest: $\{\{a,b,c,d\}\}$ [5 points], Most: $\{\{a\},\{b\},\{c\},\{d\}\}$ [5 points]

Problem 1.2.2 [3+3+4=10 points]
$R \circ R = \{(a,a),(a,d),(a,b),(a,c),(b,a),(b,b),(b,c)\}$ [3 points]
$R^{-1} = \{(b,a),(c,a),(d,c),(a,a),(a,b)\}$ [3 points]
$R$, $R \circ R$, $R^{-1}$ are not functions. [4 points]

Problem 1.3.1 [5+5=10 points]
(a) [5 points]  
(d) [5 points]  

Problem 1.3.9 [5 points]
A directed graph represents a function when there is exactly one arrow/edge leading out of each node.

Problem 1.4.1(b) [10 points]
For a given natural number n, there are only finite many ways of adding up distinct natural numbers to make n. For example, five=5=1+4=2+3. Define $N_n$ be the set of all sets of distinct natural numbers whose sum is n. Each $N_n$ is finite and thus countable. Therefore, the set of union of all $N_n$, is countable.

(ps. $N_n=\{\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{4\},\{1,3\},\{5\},\{1,4\},\{2,3\}…\}$)
Problem 1.5.9 [10 points]
Suppose set $S$ be uncountable and set $T$ be countable. Suppose $S-T$ were countable. Then $S$ would be countable because it is the union of the countable sets $S$ and $S-T$. Thus $S-T$ cannot be countable.

Problem 1.7.6(b) [5 points]
$L^+ = L^* \setminus \{e\}$ if and only if $e \notin L$.

Problem 1.8.2 [5 points]
(a) $(a \cup b)^*$

Problem 1.8.5 [5+5=10 points]
(a) true [5 points]
(c) false, $a^* b^* \cap b^* c^* = b^*$ [5 points]