CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 2

Note: 5 partial credits are given to each graded unit of problems 2.1.3, 2.1.5 and 2.2.9 for minor errors. No partial credits are given to the rest problems.

Problem 2.1.1 [5+5=10 points]
\[ e \in L(M) \text{ if and only if } s \in F. \] [5 points]

<Proof:>
Suppose \( e \in L(M) \). Then \( (s,e) \models M(q,e) \) for some \( q \in F \). But \( e \) is the empty string, so \( (s,e)=(q,e) \). Therefore \( s=q \) and thus \( s \in F \). On the other hand, if \( s \in F \), then by definition, \( e \in L(M) \). [5 points]

Problem 2.1.2 [5+5=10 points]
Top Left Machine: \( a(ba)^* \) [5 points]
Middle Right Machine: \( (ab \cup ba)^* \) [5 points]

Problem 2.1.3 [10+10=20 points]
(a) [10 points]

(b) [10 points]

Problem 2.1.5 [10 points]
Problem 2.1.7 [10 points]
By definition, \((q_0, w) \vdash^* M(q, e) \Rightarrow (q_0, w) \vdash M(q_1, a_1 w_1) \vdash M(q_2, a_1 a_2 w_2) \vdash M(q_3, a_1 a_2 a_3 w_3) \vdash \ldots \vdash M(q_n, e)\), where \(q_1, q_2, \ldots, q_n\) are reachable states and \(q_n = q\). Unreachable states will never appear in the list. Therefore, removing unreachable states from \(K\) doesn’t change the language accepted. [5 points]
Define graph \(G = (V, E)\) corresponding to a given deterministic finite automation \(M = (K, \Sigma, \delta, q_0, F)\) by \(V = K\), \((q_i, q_j) \in E\) if \((q_i, c) \vdash M(q_j, e)\) for some \(c \in \Sigma\). Run DFS (depth first search) with starting vertex \(q_0\) to find all the reachable vertices. [5 points]

Problem 2.2.1 (b) [2×5=10 points]
e, ab, abab, aba are accepted. [2×4=8 points]
abaa is rejected. [2 points]

Problem 2.2.2 [5+5=10 points]
(a) \(a^*\).
(b) \(e \cup (a(ba \cup baa)*(b \cup ba))\) or equivalently \((ab \cup aba)^*\).

Problem 2.2.9 [10+10=20 points]
(a) [10 points]

(b) [10 points]