CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 3

Note: 5 partial credits are given to each of problems 2.2.6(a), 2.3,4(c), 2.4.4, 2.3.5(a) and 2.4.10(b) for minor errors. No partial credits are given to the rest problems.

Problem 2.2.4 [5+5=10 points]
(a) Since $\triangle = \{(q_i,a_j,q_j): i \neq j\}$, once a machine is in state $q_i$, then it will never leave $q_i$. Moreover, the machine can only grant/take $a_j$ for $j \neq i$. Thus if the machine starts with state $q_i$, it accepts strings missing at least the symbol $a_i$. And in this non-deterministic finite automaton, the machine can start from any state. So it accepts the language consisting of all the strings missing at least one symbol. [5 points]
(b) Adding a new start state and transitions on $e$ from it to each of the former states, and making all the previous start states no longer initial. Then the machine has only 1 start state as normal definition and accepts the same language. [5 points]

Problem 2.2.6 [10+5+5=20 points]
(a) Nondeterministic finite automata accepting $(ab \cup aab \cup aba)^*$. [10 points]

(b) Deterministic finite automata accepting $(ab \cup aab \cup aba)^*$. [5 points]

(c) No simpler deterministic machine accepting $(ab \cup aab \cup aba)^*$ exists. It can be verified by state-minimization algorithm in section 2.5. [5 points]
Problem 2.3.1 [5+5=10 points]
(a) If we interchange the final and non-final states of a nondeterministic finite automaton, then the strings it formerly accepted are not guaranteed to be rejected. [5 points]
(b) There are examples of nondeterministic finite automata such that \((s,w) \vdash^* M(q_1,e)\) and \((s,w) \vdash^* M(q_2,e)\) for some string \(w\) and final state \(q_1\) and non-final state \(q_2\).
Then not only \(w \in L(M)\) but also the automaton resulting from interchanging start and final states accept \(w\). [5 points]

Problem 2.3.4 (c) [10 points]
\((ab)^* \cup (bc)^* ab\)

![Diagram](image-url)

Problem 2.3.5 [10+5=15 points]
(a) [10 points]

(b) [5 points]

Problem 2.3.7 (d) [5 points]
\((a \cup ba^*a)(ba^*a)^*b(b \cup a)^*\)
Problem 2.4.4 [10 points]
Suppose \( m = n = k \), then \( L = \{ w = a^k b a^k b a^{2k} : k \geq 1 \} \). Assume \( L \) is a regular expression. By theorem 2.4.1 (pumping lemma), there is an \( N \geq 1 \) such that if \( w \in L \) and \( |w| \geq N \), \( w \) can be written as \( w = xyz \) where \( y \neq \epsilon \), \( |xy| \leq N \), and \( xy^iz \) can be expressed as \( a^k b a^k b a^{2k} \) for all \( i \geq 0 \). We know \( w \) contains 2 \( bs \). Then \( y \) cannot contain any \( b \); otherwise, \( xy^0z \) has no \( b \) and \( xy^0z \notin L \). Now there are 3 possibilities: (1) \( y \) is before the first \( b \); (2) \( y \) is between the two \( bs \); and (3) \( y \) is after the second \( b \). For case (1), \( xy^2z = a^{k+|y|} b a^k b a^{2k} \notin L \) since \( k + |y| \neq k \). Contradiction. Similarly, case (2) and (3) lead to contradiction. Thus, \( L \) is not a regular expression.

Problem 2.4.8 (a) [5+5=10 points]
False. [5 points]
Each language, regular or not regular, is a subset of the regular language \( \Sigma^* \). [5 points]

Problem 2.4.10 (b) [10 points]
An example of deterministic 2-head finite automaton accepting \( wcw \) where \( w = (a \cup b)^* \)