CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 4

Note: 5 partial credits are given to each graded unit of problems 2.5.3 and 3.3.2(b) for minor errors. No partial credits are given to the rest problems.

Problem 2.5.3 [10+10=20 points]
Upper right machine of 2.1.2 [10 points]

Machine (a) of 2.2.9 [10 points]

Problem 2.6.1 (b) [5 points]
For example, $bb$ can be generated by $aa(a\cup b)^* \cup (bb)^* a^*$ but cannot be generated by $(ab\cup ba\cup a)^*$.

Problem 3.1.1 [4\times1+4\times2+3=15 points]
(a) $aa$, $baa$, $aba$, $aab$. [4\times1=4 points]
(b) $S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow bAbAb\Rightarrow baAb\Rightarrow babb\Rightarrow babbab$
$S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow bAbAb\Rightarrow bAbb\Rightarrow babbab$
$S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow bAbA\Rightarrow bAbAb\Rightarrow babb\Rightarrow babbab$
$S\Rightarrow AA\Rightarrow bAA\Rightarrow bAAb\Rightarrow bAbAb\Rightarrow babb\Rightarrow babbab$
[4\times2=8 points]
(c) $S\Rightarrow AA\Rightarrow \ldots \Rightarrow b^m AA\Rightarrow \ldots \Rightarrow b^m Ab^n A \Rightarrow \ldots \Rightarrow b^m Ab^n Ab^p \Rightarrow b^n ab^n ab^p$
[3 points]

Problem 3.1.3 (b) [5 points]
Context-free grammar for $\{ww^R : w \in \{a,b\}^*\}$:
$G = \{V, \Sigma, R, S\}$ where $V = \{a, b, S\}$, $\Sigma = \{a, b\}$, $R = \{S \rightarrow aSa, S \rightarrow bSh, S \rightarrow e\}$

Problem 3.1.5 (a) [5 points]
$S \rightarrow aB \Rightarrow abS \Rightarrow abaB \Rightarrow ababS \Rightarrow ababbA \Rightarrow ababba$

Problem 3.1.9 (f) [5 points]
Context-free grammar for $\{a^m b^n : m \leq 2n\}$:
$G = \{V, \Sigma, R, S\}$ where $V = \{a, b, A, S\}$, $\Sigma = \{a, b\}$, $R = \{S \rightarrow AA b, S \rightarrow e, A \rightarrow a, A \rightarrow e\}$.
Problem 3.2.2 [5+5=10 points]
\[ S \Rightarrow (S) \Rightarrow () \]
\[ S \Rightarrow SS \Rightarrow S(S) \Rightarrow S() \Rightarrow () \]
Thus there are two ways to derive (). Thus the context-free grammar is ambiguous. [5 points]
Example of unambiguous grammar: \( G = \langle \{(),S,T\}, \{(),\},R,S \rangle \) where \( R = \{ S \rightarrow T, S \rightarrow e, T \rightarrow (T), T \rightarrow TT \} \). [5 points]

Problem 3.2.4(b) [5+5=10 points]
\[ \text{id} + (\text{id} \cdot \text{id}) \cdot \text{id} \quad \text{(id} \cdot \text{id} \cdot \text{id}) \]

\[ E \]
\[ T \]
\[ F \]
\[ T \]
\[ F \]
\[ \text{id} \]

Problem 3.3.1 [3×1+3×1+2×1+3×1+4=15 points]
(a) There are 3 possible computations on input \( aba \). [3×1=3 points]
\[ (s,aba,e) \vdash M(f,ba,e) \]
\[ (s,aba,e) \vdash M(s,ba,a) \vdash M(s,a,aa) \vdash M(s,e,aaa) \]
\[ (s,aba,e) \vdash M(s,ba,a) \vdash M(s,a,aa) \vdash M(f,e,aa) \]

(b) There are 3 possible computations on input \( aa \). [3×1=3 points]
\[ (s,aa,e) \vdash M(f,a,e) \]
\[ (s,aa,e) \vdash M(s,a,a) \vdash M(f,e,a) \]
\[ (s,aa,e) \vdash M(s,a,a) \vdash M(f,e,aa) \]
There are 2 possible computations on input \( abb \). [2×1=2 points]
\[ (s,abb,e) \vdash M(f,bb,e) \]
\[ (s,abb,e) \vdash M(s,bb,a) \vdash M(s,b,aa) \vdash M(s,e,aaa) \]
All \( aba \), \( aa \) and \( abb \) are not accepted.
All \( baa, bab \) and \( baaaa \) are accepted. \([3 \times 1 = 3 \text{ points}]\)

\[(s,baa,e) \vdash^* M(s,aa,a) \vdash^* M (f,a,a) \vdash^* M (f,e,e)\]
\[(s,bab,e) \vdash^* M(s,ab,a) \vdash^* M (f,b,a) \vdash^* M (f,e,e)\]
\[(s,baaaa,e) \vdash^* M(s,aaaa,a) \vdash^* M (s,aa,aa) \vdash^* M (f,aa,aa) \vdash^* M (f,a,a) \vdash^* M (f,e,e)\]

(c) \( L = \{xay : x,y \in \{a,b\}^*, |x| = |y| \} \) [4 points]

Problem 3.3.2 (b) [10 points]

Example of pushdown automata accepts \( \{a^m b^n : m \leq 2n \} \):

\( M = \{K,\Sigma, T, \Delta, q, F \} \), where \( K = \{q,r\}, \Sigma = \{a,b\}, \; T = \{a\}, \; F = \{r\} \), and

\( \Delta = \{(q,a,e),(q,aa)\), \((q,a,e),(q,a)\), \((q,e,e),(r,e)\), \((r,b,a),(r,e)\) \} \)