CMSC452 Elementary Theory of Computation, Fall 2001

Sketch of Solutions: Homework 7

Note: 5 or 10 partial credits are given to each graded unit for minor errors.

Problem 4.2.1 [15 points]
\(f(w)=ww^R\) from strings in \(\{a,b\}^*\)

Problem 4.2.2 (d) [15 points]
\(L(M)=\{a\}^*\) over \(\{a,b\}\)

Problem 4.3.3 [15 points]
Let \(R_1^1, L_1^1\) and \(R_2^1, L_2^1\) denote the movement of the first and second heads, respectively, \(a_1^1\) and \(a_2^1\) denote the reading/writing of the first and second head, also respectively. The following 2-head Turing machine computes \(f(w)=ww\). Note that \(R_2^1\) means that head 2 reads rightward until blank.

Problem 4.5.1 (a) [15 points]
The Turing machine accepts \(a^*abb*bba^*\).
Problem 4.5.3 (for union) [15 points]
Given Turing machines $M_1=\{K_1, \Sigma_1, \delta_1, s_1, \{y,n\}\}$ and $M_2=\{K_2, \Sigma_2, \delta_2, s_2, \{y,n\}\}$, define non-deterministic Turing machine $M=\{K, \Sigma, \delta, s, \{y,n\}\}$ where $K=K_1 \cup K_2 \cup \{s\}$, $\Sigma=\Sigma_1 \cup \Sigma_2$, $\delta=\delta_1 \cup \delta_2 \cup \{\delta(s, \sqcup) \rightarrow (s_1, \sqcup), \delta(s, \sqcup) \rightarrow (s_2, \sqcup)\}$, and $s \notin K_1 \cup K_2$ is a new symbol. Then $L(M)=L(M_1) \cup L(M_2)$.

Problem 4.6.1 (a) [15 points]
$S \Rightarrow ABCS \Rightarrow ABCABC \Rightarrow ABCABCBC \Rightarrow ABCABCABCT_c \Rightarrow$
$ABABCBCT_c \Rightarrow ABABCBCT_c \Rightarrow ABABCBCT_c \Rightarrow ABABCBCT_c \Rightarrow$
$ABABBCCCT_c \Rightarrow AABABBCCT_c \Rightarrow AABABBCCT_c \Rightarrow$
$AABBBBCCT_{c} \Rightarrow AABBBBCT_{c} \Rightarrow AAABBBT_{b} \Rightarrow$
$AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow$
$AAAT_{a} \Rightarrow AAAT_{a} \Rightarrow AAAT_{a} \Rightarrow AAAT_{a} \Rightarrow AAAT_{a} \Rightarrow$
$AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow$
$AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow AAABBT_{b} \Rightarrow$

Problem 4.6.2 (c) [10 points]
The grammar $G=\{V, \Sigma, R, S\}$ accepts $a^n^2$, where
$V=\{a, S, T, B, C, S\}$
$\Sigma=\{a\}$
$R=\{S \rightarrow STS, T \rightarrow BTC, T \rightarrow e, BC \rightarrow CaB, aC \rightarrow Ca, Ba \rightarrow aB, $C \rightarrow $S, B \rightarrow $S, $ \rightarrow e\}$
Idea: How many swaps does bubble sort take to sort $B^nC^n$ to be $C^nB^n$? The answer is $n^2$. In this rule set, an $a$ is generated for swapping $BC$, and swapping $Ba$ or $aC$ is free (i.e. No $a$ is generated). Thus, there are $n^2$ as are generated with $B^nC^n$. 