Spring 2001

CMSC 250: Midterm 1

On this exam you are allowed one $8\frac{1}{2} \times 11$ inch piece of paper. You are NOT allowed: calculators, magnifying glasses, slide rules, PDAs or computers, textbooks, more than one page of notes, or telephones. TURN OFF ALL PHONES AND BEEPERS BEFORE THE EXAM BEGINS.

0. (5 points) Write your name, student ID number, AND section number on the front cover NEATLY.

0.5 (0 points)
We would like to post the grades on the web page so you can make sure that you know how you are doing in the class. However, we are not allowed to do anything that will associate your grades with your name or your Student ID. Therefore, we are asking for an alias, sort of a nickname that we can associate with your grades. If you do not want your grades posted at all, put “NONE” as your alias. (If you WANTED “NONE” as your alias, well, tough.)

1. (10 points) Use the following abbreviations:

a: Alice
b: Bob
S(x): student x studied for the 250 midterm.
B(x,y): student x’s score on the midterm is better than student y’s.

Translate the following from English into logical notation. Assume that the domain of discourse is the set of CMSC 250 students. You do not need to write the domain explicitly.

(a) Alice studied for 250 but Bob did not.
(b) Everyone who studied for 250 did better than Bob.
(c) There is a student with the worst score.
(d) For every student, studying was a necessary condition for scoring better than Alice.
(e) For every student who studied, there is another student who scored worse than them.
2. (14 points) Consider the following truth table:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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<td>F</td>
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</tr>
</tbody>
</table>

(a) Construct a logical statement that would have this truth table. DO NOT REDUCE.
(b) Construct a circuit that would have this truth table. Again DO NOT REDUCE.

3. (10 points) Consider the proposed proof rule, below. Is it valid? Prove or disprove using a truth table. Circle critical rows.

\[ P \rightarrow \sim (R \lor Q) \]

\[ \sim (P \rightarrow Q) \]

\[ \vdash \sim R \land \sim Q \]

4. (15 points) Use the proof rules to prove that \([A \rightarrow (B \land C)] \leftrightarrow [(B \rightarrow \sim C) \rightarrow \sim A]\) is a theorem (true with no assumptions).

5. (15 points) Assume that the domain of discourse is non-empty.
   Given:
   \[ \forall x[\sim A(x) \rightarrow K(x)] \]
   \[ \exists y[\sim K(y)] \]
   Prove: \[ \exists w[A(w) \lor \sim L(w)] \]

6. (15 points) Assume that the domain of discourse is non-empty.
   Given:
   \[ \forall a \forall b[S(a, b) \leftrightarrow (\forall x[E(x, a) \rightarrow E(x, b)])] \]
   \[ \forall a \forall b \forall c[N(a, b, c) \rightarrow (\forall x[E(x, c) \rightarrow (E(x, a) \land E(x, b))])] \]
   Prove: \[ \forall a \forall b \forall c[N(a, b, c) \rightarrow S(c, a)] \]

7. (16 points) For each statement below, give an interpretation \( P \) which makes the statement true, and another one which makes it false. The domain of discourse is \( \mathbb{Z} \). For example, \( P(x, y) \) could mean \( x < y \).
   (a) \[ \exists x[P(x, x)] \]
   (b) \[ \forall x \forall y[P(x, y) \lor P(y, x)] \]
   (c) \[ \exists x \forall y[P(x, y)] \]
   (d) \[ \forall x \exists y[P(x, y)] \]

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