Spring 2001

CMSC 250: PRACTICE Midterm 1

On this exam you are allowed one $8\frac{1}{2} \times 11$ inch piece of paper. You are NOT allowed: calculators, magnifying glasses, slide rules, PDAs or computers, textbooks, more than one page of notes, or telephones. TURN OFF ALL PHONES AND BEEPERS BEFORE THE EXAM BEGINS.

0. Write your name, student ID number, AND section number on the front cover NEATLY. (In the actual exam, you will LOSE 5 points for not writing all three pieces of information correctly).

1. Use the following abbreviations:

   A: Alice is a good driver.
   B: Ben is a good driver.
   S: Alice drives a stick-shift.
   H: Ben drives a stick-shift.

(a) Translate the following from English into logical notation:
   i. Alice and Ben both drive a stick-shift.
   ii. Ben drives a stick-shift if and only if he is a good driver, but Alice is not a good driver.

(b) Translate the following statements from logical notation into English, and provide a truth table for them.
   i. $\sim H \rightarrow \sim S$
   ii. $(B \lor H) \land \sim S$

2. Use the following abbreviations:

   a: Alice
   b: Ben
   c: Chris
   D(x): x is a good driver.
   S(x): x drives a stick-shift.
   B(x,y): x is a better (or equal) driver than y.

Assume that the domain of discourse is the set of all people. You do not need to write the domain explicitly.

(a) Translate the following from English into logical notation:
   i. Ben is the best (non stick-shift) driver among those who do not drive a stick-shift.
   ii. Neither Alice nor Chris drive a stick-shift, but both are good drivers.

(b) Translate the following from logical notation into English.
   i. $\forall x. [\sim S(x) \rightarrow \exists y. [S(y) \land B(y,x)]]$
   ii. $\exists x. [D(x) \land \sim S(x) \land B(x,b)]$
3. Assume that $x, y,$ and $z$ are input bits. Devise and draw a circuit (with two outputs, $b_1$ and $b_0$, with $b_1$ being the most significant bit and $b_0$ being the least significant bit) that outputs the function $x + y + z$. Do not simplify the circuit (i.e., write it in disjunctive normal form). You may use gates of arbitrary fan-in (i.e., arbitrary number of inputs). (Clearly label all gates, as well as inputs and outputs).

4. Given:
\[
\sim L \lor (\sim Z \lor \sim U) \\
(U \land G) \lor H \\
Z
\]
Prove: $L \rightarrow H$
[You must provide justifications and step numbers.]

5. Given:
\[
F \rightarrow (\sim G \lor H) \\
F \rightarrow G \\
\sim (H \lor I)
\]
Prove: $F \rightarrow \sim J$
[You must provide justifications and step numbers.]

6. Assume that the domain of discourse is non-empty.

Given:
\[
\forall x \forall y, [(A(x) \land B(y)) \rightarrow C(x, y)] \\
\exists y, [E(y) \land \forall w, (H(w) \rightarrow C(y, w))] \\
\forall x \forall y \forall z, [(C(x, y) \land C(y, z)) \rightarrow C(x, z)] \\
\forall w, [E(w) \rightarrow B(w)]
\]
Prove: $\forall z \forall w, [(A(z) \land H(w)) \rightarrow C(z, w)]$
[You must provide justifications and step numbers.]

7. Assume that the domain of discourse is non-empty.

Given:
\[
\forall x \forall y \forall z, [R(x, y, z) \rightarrow R(z, x, y)] \\
\forall w \forall x \forall y \forall z, [(R(x, y, z) \land R(y, z, w)) \rightarrow (R(x, z, w) \land R(x, y, w))] \\
\forall x, [\exists y (R(x, y, x) \lor R(y, x, x)) \rightarrow R(x, x, x)]
\]
Prove: $\forall x [\exists y (R(x, y, x) \lor R(y, x, x)) \rightarrow R(x, x, x)]$
[You must provide justifications and step numbers.]

(NOTE: The original problem had the first premise as: $\forall x \forall y \forall z, [R(x, y, z) \rightarrow R(z, y, x)]$. However, it may or may not work. Give it a try, and see what happens. Don’t worry, the actual exam will be “debugged” and we won’t have a note like this).