In this set of answers, I have taken the value to an actual integer. You will not need to do this on the exam for these larger numbers because calculators are not allowed, and I don’t want to spend time testing if you know how to multiply. I have done it here so that if you used a different method, it should still evaluate to reach the same integer value.

1. Answer the following about the sequence of numbers between 5 and 125 (inclusive):
   a. How many integers are between 5 and 125 (inclusive)?
      ANSWER: $125 - 5 + 1 = 121$
   b. How many integers are between 5 and 125 (inclusive) which are divisible by 3?
      ANSWER: $2 \times 3, 3 \times 3, \ldots, 40 \times 3, 41 \times 3 = 6, 9, \ldots, 120, 123$
      $41 - 2 + 2 = 40$
   c. How many integers are between 5 and 125 (inclusive) which are divisible by 5?
      ANSWER: $1 \times 5, 2 \times 5, \ldots, 24 \times 5, 25 \times 5 = 5, 10, \ldots, 120, 125$
      $25 - 1 + 1 = 25$
   d. How many integers are between 5 and 125 (inclusive) which are divisible by both 3 and 5?
      ANSWER: $1 \times 15, 2 \times 15, \ldots, 7 \times 15, 8 \times 15 = 15, 30, 105, 120$
      $8 - 1 + 1 = 8$
   e. How many integers are between 5 and 125 (inclusive) which are divisible by either 3 or 5?
      ANSWER: $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 40 + 25 - 8 = 57$
   f. How many integers are between 5 and 125 (inclusive) which are divisible by neither 3 nor 5?
      ANSWER: Total number - number that are divisible $= n(U) - n(A \cup B) = 121 - 57 = 64$
   g. How many integers are between 5 and 125 (inclusive) which are divisible by 3 but not divisible by 5?
      ANSWER: number divisible by 3 - number of those that are divisible by 5 $= n(A) - n(A \cap B) = 40 - 8 = 32$

2. For the next set of questions, assume you have a of 6 dogs. Some dogs are big (marked with a b), some are middle sized (marked with a m), and some are small (marked with an s). These dogs are {Alpi (b), Bingo (s), Congo (b), Delfi(s), Elf (s), Fred (m)}. Answer the following questions about the dogs in your kennel.
   a. How many different ways can you select two dogs to take for a walk at the same time?
      ANSWER: $\binom{6}{2} = \frac{6!}{2!4!} = 15$
   b. How many different ways can you select two dogs to take for a walk at the same time assuming you can’t handle 2 big dogs at the same time?
      ANSWER: Partition and use Addition Rule
      Two non-big dogs + One big and one non-big dog
      $\binom{4}{2} + \binom{3}{1} \binom{1}{1} = 6 + 8 = 14$
   c. How many different ways can you assign the dogs all to leashes (assuming you have 6 leashes in 6 different colors)?
      ANSWER: Steps and use multiplication rule
      Select the leash for the first dog = 6
      * Select the leash for the second dog = 5
      * ...
      * Select the leash for the last dog = 1
      $= 6!$
d. How many different ways can you assign the dogs all to leashes (assuming you have 6 leashes in 6 different colors), but also assuming you have only two leashes that can handle the big dogs?

**ANSWER:** Steps and use the multiplication rule

1. Select the leashes for the big dogs
2. Select the leashes for the other dogs

$$= 2! * 4! = 2 * 24 = 48$$

e. How many different ways can you divide the dogs up for walking among your 3 volunteers – assuming each volunteer must walk 2 dogs each?

**ANSWER:** Steps and use the multiplication rule

1. Select the dogs for the first walker
2. Select the dogs for the second walker
3. Select the dogs for the third walker

$$= \binom{6}{2} \binom{4}{2} = 15 * 6 * 1 = 90$$

f. How many different ways can you line up the dogs (single file) for a dog show?

**ANSWER:** Linear permutation of all 6 dogs = 6! = 720

g. How many different ways can you line up the dogs (single file) for a dog show assuming the large dogs must come first, followed by the medium then the small dogs?

**ANSWER:** Multiplication rule of the Linear permutations for the individual sizes

$$= 2! * 1! * 3! = 2 * 1 * 6 = 12$$

h. How many different ways can you create a line in the dog show of four finalists?

**ANSWER:** r-Permutation : $$P(6, 4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{720}{2} = 360$$

i. How many different ways can you distribute the 15 doggie treats you have (assuming the doggie treats are indistinguishable)? (note: it can be that one dog gets all of the treats - you are not trying to distribute them evenly.)

**ANSWER:** 15 indistinguishable doggie treats into 6 categories

$$\frac{(15 + (6-1))!}{15!(6-1)!} = \frac{20!}{15!5!} = 15504$$

j. How many different ways can you assign them to the 4 run areas at your kennel? (note: these “runs” are distinguishable, but you could be assigning them all to the same run or as evenly as possible.)

**ANSWER:** 6 distinguishable dogs into 4 distinguishable runs

- Dog 1 has 4 runs to select from, dog 2 has 4 runs to select from ...
- Dog 6 has 4 runs to select from

$$= 4^6 = 3096$$

k. Assuming you have 6 collars (6 different colors) and 6 leashes (6 different colors), how many ways can you assign the collars and leashes to your six dogs so they can go for a walk?

**ANSWER:** Use the multiplication rule with the two individual steps of assigning collars and then assigning leashes.

$$= 6! * 6! = 720^2 = 518400$$

l. Assuming you have 5 families interested in adopting dogs, how many different ways can the dogs be given to those families? (note: you are assuming there is no personal preference taken into account.)

**ANSWER:** r-Permutation of assigning 5 of the 6 dogs to distinguishable families

$$= P(6, 5) = \frac{6!}{(6-5)!} = \frac{6!}{1!} = 6! = 720$$

m. Assuming all of your dogs are male, and the neighbor’s dog turns out to be pregnant, how many different ways could your dogs be the father of her puppies?

**ANSWER:** Select the one of your dogs that is the father = \(\binom{6}{1} = 6\)

If you want to assume it may be another dog from somewhere else (none of yours are the father this is just one more possibility = \(\binom{1}{1} = 7\)
n. Assuming you must take one dog of each size with you, how many different ways do you have to select your companions?

ANSWER: select the one dog from each set and use the multiplication rule to get the whole

\[ \binom{2}{1} \binom{1}{1} \binom{3}{1} = 2 \times 1 \times 3 = 6 \]