CMSC 250   Exam #2 - ver a ANSWERS   Thurs., Nov. 20, 2003

Write all answers legibly on the paper provided. If you need extra paper, raise your hand and request a blank paper – you must put your name on and hand-in any paper you receive. You can also use the back of the last page which is blank. Clearly label any answers that appear on a paper different from where the question appears. You must indicate the continuation of the answer on the paper where the question is and on the paper where the answer is continued. The number of points possible for each question is indicated in square brackets – the total number of points on the exam is 125, and you will have exactly 1 hour and 45 minutes to complete this exam. In order to receive any partial credit, you must show your work, clearly labeled in the space provided. You may not use calculators, textbooks or any other external aids during this exam. The formula sheet is attached - this can be removed from the back of the exam and does not need to be handed in at the end.

Write the following University approved honor pledge and sign on the blank provided after you complete the exam. This can not be done after we make the “stop writing now” announcement.

I pledge on my honor that I have not given or received any unauthorized assistance on this examination.

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Sign Here: ________________________________

**** This area is for grading purposes (points lost per page)- Do not write below this line ****

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1. [20 pnts.] Answer each of the following questions.

You do not have to do the arithmetic, you can leave the answer in a format that includes combinations using the format \( \binom{x}{y} \), permutations using the format \( P(x, y) \), addition, subtraction, multiplication, division, exponents and/or factorials.

a. How many distinct arrangements can be formed using the letters of “tallahassee”? (Note: arrangement means - it must use all of the characters present, but two occurrences of the same character are indistinguishable)

ANSWER: \( \frac{11!}{3!2!2!2!} = \binom{11}{2}\binom{9}{3}\binom{6}{2}\binom{4}{2} \)

b. In a multiple choice test with 35 questions in which each of the questions has five possible answers (a,b,c,d,e), what is the probability that the answers to the test have the same number of questions where the answer is each of the letters? (In other words, what is the probability that the number of questions having 'a' as the correct answer is the same as the number of questions where 'b' is the answer and so on for all 5 possible answers.)

ANSWER: \( \frac{35!}{(7!)^5} = \binom{35}{7}\binom{28}{7}\binom{21}{7}\binom{14}{7} \)

c. In that same multiple choice test from the previous question, what is the probability that you will get 100% on the exam if you know (for a fact) that there are the same number of each letter on the answer key? (In other words, information was given that there would be the same number of questions with an 'a' as the answer as questions with a 'b' as the answer, etc., what is your probability of getting them all right?) Assume you are guessing randomly to answer the questions on the multiple choice test.

ANSWER: \( \frac{1}{\binom{35}{7}} \)

d. Assume you have in your pocket 2 quarters (25 cents each), 5 dimes (10 cents each), and 3 nickels (5 cents each). You reach into your pocket and randomly remove 2 coins. What is the probability that you have exactly 30 cents in your hand?

ANSWER: \( \frac{6}{\binom{10}{2}} \)

e. In a shipment of five automobiles, one is defective. Two people come to purchase automobiles. What is the probability that the second car purchased is not defective given that the first car purchased was not defective?

ANSWER: \( \frac{3}{4} \)
2. [35 pnts.] For each of the four parts of this question: Either prove the statement or give a counter example to disprove. If you are disproving, you must give specific members for the sets A, B, C, D and U (the universal set) as needed or prove the following statements concerning Sets. Note: \( n(X) \) is the function used to indicate the size of set X.

Be sure to give the name of the reason which justifies each step you give in the proof - the only rules you may assume are those on the sheet attached.

a. \( \forall A, B, C \in \{\text{sets}\}, (A \subseteq C) \rightarrow [(A \cap B) \subseteq (C \cap (B \cup A))] \)

Let A, B, and C be arbitrary within \( \{\text{sets}\} \).

Assume \( A \subseteq C \)

Let \( X \) be arbitrary within the Universe

Assume \( X \in A \cap B \)
\( X \in A \land X \in B \) by definition of intersection
\( X \in A \) by conjunctive simplification
\( X \in C \) by definition of subset since \( A \subseteq C \)
\( X \in B \lor X \in A \) by disjunctive addition
\( X \in B \cup A \) by definition of Union
\( X \in C \land X \in B \cup A \) by conjunctive addition
\( X \in (C \cap (B \cup A)) \) by definition of intersection

\( X \in A \cap B \rightarrow X \in C \cap (B \cup A) \) by closing the conditional world without contradiction
\( \forall X \in \text{Universe}, X \in A \cap B \rightarrow X \in X \cap (B \cup A) \) by generalizing from the generic particular
\( A \cap B \subseteq C \cap (B \cup A) \) by definition of subset
\( A \subseteq C \rightarrow A \cap B \subseteq C \cap (B \cup A) \) by closing conditional world without contradiction
\( \forall A, B, C \in \{\text{sets}\}, A \subseteq C \rightarrow A \cap B \subseteq C \cap (B \cup A) \) by generalizing from the Generic Particular
Q.E.D.
b. \( \forall A, B, C \in \{\text{sets}\}, (A - B) \times (C - B) = (A \times C) - B \)

**Answer:** THIS IS FALSE

Let \( A = \{1\} \) and \( B = \{1\} \) and \( C = \{1\} \)

\[
A - B = \emptyset \\
C - B = \emptyset \\
(A - B) \times (C - B) = \emptyset \times \emptyset = \emptyset \\
A \times C = \{(1, 1)\} \\
(A \times C) - B = \{(1, 1)\} - \{1\} = \{(1, 1)\}
\]

Since \( \emptyset \neq \{(1, 1)\} \), this equality does not hold and this is a valid counter example.
c. \( \forall A, B, C \in \{ \text{sets} \}, ((C - B) \cap A) \cup (A - (B \cup C)) = A - B \)

\(((C - B) \cap A) \cup (A - (B \cup C))\) is the left hand side

\[= ((C \cap B') \cap A) \cup (A \cap (B \cup C)') \text{ by alt rep of set diff} \]

\[= (A \cap (C \cap B')) \cup (A \cap (B \cup C)') \text{ by commutativity} \]

\[= A \cap ((C \cap B') \cup (B \cup C)') \text{ by distribution} \]

\[= A \cap ((C \cap B') \cup (B' \cap C')) \text{ by DeMorgan's Law} \]

\[= A \cap ((C \cup C') \cap B') \text{ by Distribution} \]

\[= A \cap (U \cap B') \text{ by union with comp.} \]

\[= A \cap B' \text{ by intersection with the universe} \]

\[= A - B \text{ by alt rep of set diff} \]

Q.E.D.
d. \( \forall A, B, C, D \in \{\text{sets}\}, (A - B) \times (C - D) \subseteq (A \times C) \cup (B \times D) \)

**ANSWER:**

Let A, B, C, and D be arbitrary in \( \{\text{sets}\} \)

Let \((x, y)\) be arbitrary in the Universe

Assume \((x, y) \in (A - B) \times (C - D)\)

\(x \in (A - B) \land y \in (C - D)\) by definition of Cartesian Product

\(x \in A \land A \not\in B \land y \in C \land y \not\in D\) by def of set diff

\(x \in A \land y \in C\) by conjunctive simplification

\((x, y) \in A \times C\) by definition of Cartesian Product

\((x, y) \in A \times C \lor (x, y) \in B \times D\) by disjunctive addition

\((x, y) \in (A \times C) \cup (B \times D)\) by definition of Union

\(\forall (x, y) \in \text{Universe}, (x, y) \in (A - B) \times (C - D) \rightarrow (x, y) \in (A \times C) \cup (B \times D)\) by closing the conditional world and generalizing from the GP \((A - B) \times (C - D) \subseteq (A \times C) \cup (B \times D)\) by definition of subset

\(\forall A, B, C, D \in \{\text{sets}\}, (A - B) \times (C - D) \subseteq (A \times C) \cup (B \times D)\) by generalizing from the GP

Q.E.D.
3. [55 pts.] For all of the parts of this question: Either find a specific counter example or prove that
the statement is true using a formal proof method. When using induction to prove something true,
you must only use strong induction if it is required by that problem – using strong induction to prove
something that only required regular induction, will result in a loss of points.

a. \( \forall n \in \mathbb{Z} \geq 1, \ 4 \mid (5^{2n} + 3^{4n-1}) \)

**ANSWER:**

**Base Case:** \( (n = 1) \)

\[
5^{2\cdot1} + 3^{4\cdot1-1} = 5^2 + 3^3 = 25 + 27 = 52
\]

\( 4 \mid 52 \)

**Inductive Hypothesis:** \( (n=k) \)

\( 4 \mid 5^{2k} + 3^{4k-1} \)

**Inductive Step:** \( (n=k+1) \)

**show:**

\( 4 \mid 5^{2(k+1)} + 3^{4(k+1)-1} \)

**proof:**

\[
5^{2(k+1)} + 3^{4(k+1)-1} = 5^{2k+2} + 3^{4k+3}
\]

\[
= 5^{2k} \cdot 5^2 + 3^{4k-1} \cdot 3^4
\]

\[
= 25(5^{2k}) + 81(3^{4k-1})
\]

\[
= 25(5^{2k}) + (25 + 56)(3^{4k-1})
\]

\[
= 25(5^{2k}) + 3^{4k-1} + 56(3^{4k-1})
\]

Since by the IH, \( 4 \mid 5^{2k} + 3^{4k-1} \), \( \exists m \in \mathbb{Z} \) such that \( 5^{2k} + 3^{4k-1} = 4m \). So Substituting in to
the previous statement:

\[
= 25(4m) + 56(3^{4k-1})
\]

\[
= 4(25m + 14(3^{4k-1})
\]

Since \( 25m + 14(3^{4k-1}) \in \mathbb{Z} \) by closure of \( \mathbb{Z} \) in addition, multiplication, and positive integer
exponents, we know that

\( 4 \mid 5^{2(k+1)} + 3^{4(k+1)-1} \) by the definition of divides

QED
b. Suppose that \( h_0, h_1, \ldots \) is a sequence defined as follows:
\[
h_0 = 1, \ h_1 = 2, \ h_2 = 3 \\
h_k = h_{k-1} + h_{k-2} + h_{k-3} \text{ for all } k \geq 3
\]

Prove or give a counterexample to the statement \( \forall n \in \mathbb{Z}^0, h_n \leq 3^n \)

**ANSWER:**

**Base Case:** (n = 0, 1 and 2)
\[
\begin{align*}
n=0: & \quad h_0 = 1, \ 3^0 = 1, \text{ and } 1 \leq 1 \\
n=1: & \quad h_1 = 2, \ 3^1 = 3, \text{ and } 2 \leq 3 \\
n=2: & \quad h_2 = 3, \ 3^2 = 9, \text{ and } 3 \leq 9
\end{align*}
\]

**Inductive Hypothesis:** (n=i \( \forall i \in \mathbb{Z} \text{ where } 3 \leq i < p \))
h_i \leq 3^i \text{ where } h_i = h_{i-1} + h_{i-2} + h_{i-3}

**Inductive Step:** (n=p)

**show:**
h_p \leq 3^p

**proof:**
\[
\begin{align*}
h_p &= h_{p-1} + h_{p-2} + h_{p-3} \\
h_p &\leq 3^{p-1} + 3^{p-2} + 3^{p-3} \text{ by the IH} \\
h_p &\leq 3^p(3^{-1} + 3^{-2} + 3^{-3}) \text{ by distribution} \\
h_p &\leq 3^p(\frac{1}{3} + \frac{1}{9} + \frac{1}{27}) \\
h_p &\leq 3^p(\frac{13}{27}) \\
h_p &\leq 3^p(\frac{13}{27})
\end{align*}
\]

Since \( \frac{13}{27} \) is a real between 0 and 1, \( 3^p(\frac{13}{27}) \leq 3^p \)

And by transitivity, \( h_p \leq 3^p \)

QED

----------OR----------

\[
\begin{align*}
h_p &= h_{p-1} + h_{p-2} + h_{p-3} \\
h_p &\leq 3^{p-1} + 3^{p-2} + 3^{p-3} \text{ by the IH} \\
3^{p-2} &\leq 3^{p-1} \text{ and } 3^{p-3} \leq 3^{p-1} \text{ by comparing positive exponents on a positive base}
\end{align*}
\]

\[
\begin{align*}
h_p &\leq 3^{p-1} + 3^{p-1} + 3^{p-1} \text{ by substitution} \\
h_p &\leq 3(3^{p-1}) \\
h_p &\leq 3^p \\
QED
\]
c. \( \forall n \in \mathbb{Z}_{\geq 1}, \sum_{j=0}^{n} (n - j)2^j = 2^{n+1} - n - 2 \)

ANSWER:

**Base Case:** \((n = 1)\)
\[
\sum_{j=0}^{1} (1 - j)2^j = (1 - 0)2^0 + (1 - 1)2^1 = 1 + 0 = 1
\]
\[
2^{1+1} - 1 - 2 = 2^2 - 3 = 4 - 3 = 1
\]
\[= 1\]

**Inductive Hypothesis:** \((n=k)\)
\[
\sum_{j=0}^{k} (k - j)2^j = 2^{k+1} - k - 2
\]

**Inductive Step:** \((n=k+1)\)

show:
\[
\sum_{j=0}^{k+1} ((k + 1) - j)2^j = 2^{(k+1)+1} - (k + 1) - 2
\]

proof:
\[
\sum_{j=0}^{k+1} ((k + 1) - j)2^j
\]
\[
= \sum_{j=0}^{k+1} ((k - j) + 1)2^j
\]
\[
= \sum_{j=0}^{k+1} (k - j)2^j + 2^j
\]
\[
= \sum_{j=0}^{k+1} (k - j)2^j + \sum_{j=0}^{k+1} 2^j
\]
\[
= \sum_{j=0}^{k} (k - j)2^j + \sum_{j=k+1}^{k+1} (k - j)2^j + \sum_{j=0}^{k+1} 2^j
\]
\[
= 2^{k+1} - k - 2 + \sum_{j=k+1}^{k+1} (k - j)2^j + \sum_{j=0}^{k+1} 2^j \text{ by the IH}
\]
\[
= 2^{k+1} - k - 2 + (k - (k + 1))2^{k+1} + \sum_{j=0}^{k+1} 2^j \text{ by evaluating the summation}
\]
\[
= 2^{k+1} - k - 2 + (k - (k + 1))2^{k+1} + \frac{2^{k+1+1} - 1}{2-1} \text{ by Theorem 4.2.3}
\]
\[
= 2^{k+1} - k - 2 + (k - (k + 1))2^{k+1} + 2^{k+1+1} - 1
\]
\[
= 2^{k+1} - k - 2 - 2^{k+1} + 2^{k+2} - 1
\]
\[
= -k - 2 + 2^{k+2} - 1
\]
\[
= 2^{k+2} - k - 1 - 2
\]
\[
= 2^{k+2} - (k + 1) - 2
\]

QED
4. [15 pnts.] A doughnut shop sells 20 types of doughnuts. Two doughnuts of the same type are not distinguishable from each other. These types of doughnuts are then divided into 3 categories to make it easier for you to find the type you want. The 3 categories are chocolate, fruit filled and frosted. 10 of the types are classified as chocolate, 7 of them are fruit filled and 3 of them are frosted (each type of doughnut fits only into one of these categories). The doughnut shop has available 5 of each type of doughnut. You and 3 friends (all of which are distinguishable from each other) go to the doughnut shop. Answer the following questions about your trip assuming the events mentioned in one part of this question do not affect the other parts of this question.

NOTE: You do not have to do all of the arithmetic, but you do have to get each answer to a form that includes ONLY: addition, subtraction, multiplication, division, exponents and factorials.

a. You each pick one doughnut. (Reminder: the group includes you and 3 of your friends.) What is the probability that no two people picked doughnuts of the same type?

   ANSWER: \( \frac{20 \times 19 \times 18 \times 17}{20^4} \)

b. Again, assuming you each pick one doughnut, what is the probability that two or more people have picked doughnuts from the same type? (Remember: the individual parts to this question do not affect each other.)

   ANSWER: 1 - (previous answer) = 1 - \( \frac{20 \times 19 \times 18 \times 17}{20^4} \)

c. Again, assuming you each pick one doughnut, what is the probability that no two people picked doughnuts from the same category?

   ANSWER: 0
d. Assume you and all 3 of your friends are very fond of chocolate and none of you would consider buying anything except a doughnut classified as “chocolate”. You each pick one doughnut. What is the probability that each of the four doughnuts chosen is of a different type?
   ANSWER: \( \frac{10 \times 9 \times 8 \times 7}{10^4} \)

e. You decide these doughnuts are so wonderful that you and your friends must buy all 100 of the available doughnuts. So you buy them, but you don’t want to make yourself sick by eating that many doughnuts so you decide to do some backwards trick-or-treating. You put one doughnut in each of the mailboxes along your street (you may assume there are exactly 100 mailboxes in a line on this street). How many ways can you distribute the doughnuts into the mailboxes taking into account that two doughnuts of the same type are not distinguishable?
   ANSWER: \( \frac{100!}{(5!)^{20}} \)

f. The doughnut shop likes your business so much that they make you and your friends 50 identical chocolate doughnuts. How many ways are there for you to distribute those doughnuts between the four of you assuming you do not have to distribute them evenly?
   ANSWER: \( \frac{(50+4-1)!}{50!(4-1)!} \)
This page intentionally blank for you to have extra paper to continue any answer that took more room than the space provided.
Theorem 1.1.1 - Epp Textbook p. 14

Given any statement variables $p$, $q$, and $r$, a tautology $t$ and a contradiction $c$, the following logical equivalences hold:

1. Commutative laws: $p \land q \equiv q \land p$
   $p \lor q \equiv q \lor p$

2. Associative laws: $(p \land q) \land r \equiv p \land (q \land r)$
   $(p \lor q) \lor r \equiv p \lor (q \lor r)$

3. Distributive laws: $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
   $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

4. Identity laws: $p \land t \equiv p$
   $p \lor c \equiv p$

5. Negation laws: $p \lor \sim p \equiv t$
   $p \land \sim p \equiv c$

6. Double Negation law: $\sim (\sim p) \equiv p$

7. Idempotent laws: $p \land p \equiv p$
   $p \lor p \equiv p$

8. DeMorgan’s laws: $\sim (p \land q) \equiv \sim p \lor \sim q$
   $\sim (p \lor q) \equiv \sim p \land \sim q$

9. Universal bounds laws: $p \lor t \equiv t$
   $p \land c \equiv c$

10. Absorption laws: $p \lor (p \land q) \equiv p$
    $p \land (p \lor q) \equiv p$

11. Negations of $t$ and $c$: $\sim t \equiv c$
    $\sim c \equiv t$

Table 1.3.1 - Epp Textbook p. 39

<table>
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<tr>
<th>Modus Ponens</th>
<th>Modus Tollens</th>
<th>Disjunctive Syllogism</th>
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<td>$p \rightarrow q$</td>
<td>$\sim q$</td>
<td>Therefore $p$</td>
<td>$\sim p$</td>
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Table 5.2.1 - Subset Relations

Given any sets \(A, B, \text{ and } C:\)

1. Inclusion for Intersection:
   \[(A \cap B) \subseteq A \quad (A \cap B) \subseteq B\]

2. Inclusion for Union:
   \[A \subseteq (A \cup B) \quad B \subseteq (A \cup B)\]

3. Transitive Property of Subsets:
   \[(A \subseteq B) \land (B \subseteq C) \rightarrow A \subseteq C\]

Theorem 5.2.2 - Set Identities

Given any sets \(A, B, \text{ and } C, \) the universal set \(U\) and the empty set \(\emptyset:\)

1. Commutative laws:
   \[A \cap B = B \cap A \quad A \cup B = B \cup A\]

2. Associative laws:
   \[(A \cap B) \cap C = A \cap (B \cap C) \quad (A \cup B) \cup C = A \cup (B \cup C)\]

3. Distributive laws:
   \[A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\]

4. Intersection with \(U\) (Identity):
   \[A \cap U = A\]

5. Double Complement law:
   \[(A')' = A\]

6. Idempotent laws:
   \[A \cap A = A \quad A \cup A = A\]

7. De Morgan’s laws:
   \[(A \cup B)' = A' \cap B' \quad (A \cap B)' = A' \cup B'\]

8. Union with \(U\) (Universals Bounds):
   \[A \cup U = U\]

9. Absorption laws:
   \[A \cup (A \cap B) = A \quad A \cap (A \cup B) = A\]

10. Alternative Representation for Set Diff:
    \[A - B = A \cap B'\]

Table 5.2.3 - Subset Intersection and Union

Given any sets \(A, \text{ and } B:\)

1. \(A \subseteq B \rightarrow (A \cap B = A)\) Intersection with Subset
2. \(A \subseteq B \rightarrow (A \cup B = B)\) Union with Subset

Table 5.3.3 (plus others) - Properties of \(\emptyset\) and Universal set

Given any sets \(A, B, \text{ and } C, \text{ the universal set } U\) and the empty set \(\emptyset:\)

1. Union with \(\emptyset\):
   \[A \cup \emptyset = A\]

2. Intersection and Union with Complement
   \[A \cap A' = \emptyset \quad A \cup A' = U\]

3. Intersection with \(\emptyset\):
   \[A \cap \emptyset = \emptyset\]

4. Complement of Union and \(\emptyset\):
   \[U' = \emptyset \quad \emptyset' = U\]

5. Every set is subset of Universal:
   \[\forall A \in \{Sets\}, A \subseteq U\]

6. Empty set is subset of every set:
   \[\forall A \in \{Sets\}, \emptyset \subseteq A\]

7. Definition of Empty Set:
   \[\forall A \in \{Sets\}, A = \emptyset \Rightarrow \forall x \in U, x \notin A\]

Things from Ch. 4

Theorem 4.1.1
\[\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)\]

Theorem 4.1.1
\[c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} (c \cdot a_k)\]

Theorem 4.1.1
\[\prod_{k=m}^{n} a_k \cdot \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k \cdot b_k)\]

Theorem 4.2.2
\[\sum_{i=1}^{m} i = \frac{m(m+1)}{2}\]

Theorem 4.2.3
\[\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}\]