CMSC250, Fall 2003

Homework 1 Answers

Due Wednesday, September 10 at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points.

1. In the game of baseball, the two middle infield positions are shortstop and second base. There must always be exactly one person playing each position at all times, and no person can play more than one position simultaneously.

Using the following definitions of propositions,

- \( p \) = “Pat is playing shortstop.”
- \( q \) = “Quentin is playing shortstop.”
- \( r \) = “Rachel is playing second base.”
- \( s \) = “Sal is playing second base.”

write a logical expression (using only \( \land, \lor, \text{and} \sim \)) which completely expresses the same meaning of each compound statement given below, taking into account the rules of baseball described above. Unless stated otherwise, you may assume there are other (unnamed) players who can play shortstop and second base, and you do not need to express those other peoples’ playing in your answers.

(a) Only Rachel and Sal can play second base, so one of them must be playing.

\[
\text{Answer: } (r \land s) \lor \sim(r \land s)
\]

(b) Assuming these are the only four players who can play shortstop and second base, express the fact that Quentin refuses to play at the same time as Rachel.

\[
\text{Answer: } (q \land s) \lor (p \land s) \lor (p \land r)
\]

(c) Assuming these are the only four players who can play shortstop and second base, express the fact that Rachel and Quentin will only play if the other is also playing.

\[
\text{Answer: } (q \land r) \lor (p \land s)
\]

(d) Assuming there are other (unnamed) players who can play shortstop and second base, express the fact that Rachel and Quentin will only play if the other is also playing.

\[
\text{Answer: } (q \land r) \lor \sim(q \lor r)
\]

2. Express the negations of each of the following statements in a normal, English sentence by applying DeMorgan’s law to the negation of each statement.

(a) The Yankees or the Mets will play tomorrow (or both).

\[
\text{Answer: } \sim(p \lor q) \
\]
**Answer:** The Yankees will not play tomorrow and the Mets will not play tomorrow. (OR) Neither the Yankees nor the Mets will play tomorrow.

(b) The flesh is willing but the spirit is weak.

**Answer:** The flesh is not willing or the spirit is not weak.

(c) This hotdog is not tasty and it cost more than $5.

**Answer:** This hotdog is tasty or it cost $5 or less.

3. Assuming x is a particular real number,

(a) write the negation of this statement: $-4 < x \leq 1$.

**Answer:** $(-4 \geq x) \lor (x > 1)$

(b) Give one possible value that x could be if the statement $-4 < x \leq 1$ is false.

**Answer:** Many. Anything less than or equal to -4 or bigger than 1 is fine.

4. State whether each of the following sentences are statements. If a sentence is a statement, state whether it is true or false.

(a) “Why can’t the Orioles beat the Devil Rays?”

**Answer:** Not a statement (it’s a question).

(b) “There is no integer that is both greater than 8 and less than 5.”

**Answer:** True statement.

(c) “This sentence is a statement.”

**Answer:** True statement.

(d) “This sentence is either false or it isn’t.”

**Answer:** True statement.

5. Let $a, b,$ and $c$ be statements. Construct three complete truth tables, one for each of the following statements:

(a) $(a \land b) \lor (\sim a \land b)$

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<tr>
<th>$a$</th>
<th>$b$</th>
<th>$a \land b$</th>
<th>$\sim a$</th>
<th>$a \land b$</th>
<th>$(a \land b) \lor (\sim a \land b)$</th>
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**Answer:**

(b) $(\sim a \land \sim b) \lor (a \land \sim b)$

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<th>$a \land \sim b$</th>
<th>$\sim a \land \sim b$</th>
<th>$(\sim a \land \sim b) \lor (a \land \sim b)$</th>
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(c) \((\sim a \lor b) \land (\sim b \lor c) \land a\)

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<th>c</th>
<th>~a</th>
<th>~b</th>
<th>~a \lor b (X)</th>
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<th>(X) \land (Y) \land a</th>
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Answer: