**Chapter 4**

**Mathematical Induction**
- Used to verify a property of a sequence
- $2, 4, 6, 8, \ldots$ for $i \geq 1$ $a_i = 2i$
  - infinite sequence with infinite distinct values
- for $i \geq 1$ $b_i = (-1)^i$
  - infinite sequence with finite distinct values
- for $1 \leq i \leq 6$ $c_i = i+5$
  - finite sequence (with finite distinct values)

**Finding the Explicit Formula**
- Figure the formula of this sequence
- $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$
- Different sequences with same initial values
  - $k \geq 0$
  - $a_k = 2k + 1$
  - $b_k = (k-1)^3 + k + 2$

**Summation & Product Notation**
- Sum of Items Specified
  $$\sum_{k=1}^{6} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$
- Product of Items Specified
  $$\prod_{k=1}^{5} 2k = 2(1) * 2(2) * 2(3) * 2(4) * 2(5)$$
Variable ending point

• n as the index of the final term
  \[ \sum_{k=0}^{n} \frac{k+1}{n+k} \]

• for n = 2
• for n = 3

Nesting of Sum/Product Notation

• Variations (same or different??):
  \[ \sum_{j=1}^{j} \sum_{i=1}^{n} Y_{ij}^2 = \sum_{j=1}^{j} (\sum_{i=1}^{n} Y_{ij})^2 = (\sum_{j=1}^{j} \sum_{i=1}^{n} Y_{ij})^2 \]

Telescoping Series

\[ \sum_{k=1}^{n} \left( \frac{k}{k+1} - \frac{k+1}{k+2} \right) \]
\[ \prod_{i=1}^{n} \left( \frac{i}{i+1} \right) \]
Properties

- Merging and Splitting

\[
\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k) \quad \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} a_k + \sum_{k=m+1}^{n} a_k
\]

\[
\prod_{k=m}^{n} a_k \cdot \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k \cdot b_k) \quad \prod_{k=m}^{n} a_k = \prod_{k=m}^{n} a_k \cdot \prod_{k=m+1}^{n} a_k
\]

- Distribution

\[c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} (c \cdot a_k)\]

Factorial

- \( n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 \)

- Definition

\[0! = 1 \]
\[n! = n \cdot (n-1)! \]