Questions?

- Projects
- Homework
Two-way ANOVA

• Setting
  – Full factorial design: \( k \) columns, \( l \) rows
  – For each cell (treatment) you gathered from a random samples
    • Mean
    • Standard deviation

• Null hypothesis
  – \( F \) ratios (row column and interaction) follow a \( F \) distribution
    • Main effect: \( F_{\text{col}} \) and \( F_{\text{row}} \)
    • Interaction: \( F_{\text{int}} \)
**F ratios**

\[ F_{\text{col}} = \frac{MS_{\text{col}}}{MS_{\text{cell}}} \]
with
\[
MS_{\text{col}} = \frac{SS_{\text{col}}}{df_{\text{col}}}, \quad SS_{\text{col}} = n_{\text{col}} \sum_{j=1}^{k} (\bar{X}_j - \bar{X}_{\text{col}})^2
\]
\[ df_{\text{col}} = \text{col\_count} - 1 \]

\[ F_{\text{row}} = \frac{MS_{\text{row}}}{MS_{\text{cell}}} \]
with
\[
MS_{\text{row}} = \frac{SS_{\text{row}}}{df_{\text{row}}}, \quad SS_{\text{row}} = n_{\text{row}} \sum_{j=1}^{l} (\bar{X}_j - \bar{X}_{\text{row}})^2
\]
\[ df_{\text{col}} = \text{row\_count} - 1 \]

\[ F_{\text{int}} = \frac{MS_{\text{int}}}{MS_{\text{cell}}} \]
with
\[
MS_{\text{int}} = \frac{SS_{\text{int}}}{df_{\text{int}}}, \quad SS_{\text{int}} = \left( \sum_{j=1}^{k-l} \frac{T_j^2}{n_j} - \frac{T^2}{N} \right) - SS_{\text{row}} - SS_{\text{col}} - SS_{\text{bet}}
\]
\[ df_{\text{int}} = df_{\text{row}} \times df_{\text{col}} \]

and
\[
MS_{\text{cell}} = \frac{SS_{\text{cell}}}{\text{cell\_count}}, \quad SS_{\text{cell}} = \sum \sigma_j^2, \quad \text{assuming equal size cells}
\]
\[ df_{\text{cell}} = N_T - \text{cell\_count} \]
Interpretation (General Linear Model)

Score = Grand mean 
+ Row effect 
+ Column effect 
+ Interaction effect 
+ Error
Interactions

From Explaining Psychological Statistic (Cohen)
Interactions (Results)

C = Column means
R = Row means

Interaction is not significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = not significant
R = not significant

Interaction is significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = not significant
R = not significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = not significant
R = significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = significant
R = not significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = not significant
R = not significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = significant
R = significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = significant
R = not significant

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>R2</td>
</tr>
</tbody>
</table>

C = significant
R = significant
Interactions (Caveats)

• If the interaction is significant
  – Main effect must be interpreted with care

• If the interaction is not significant
  – Main effect can be interpreted with 2 separate ANOVAs
Two-way ANOVA example
Other tests

• Repeated measure ANOVA
  – For within subject design

• Mixed Design ANOVA
  – For design which are both between and within subjects

• Nonparametric Statistics
  – Distribution-free tests
    • non-ordinal scale
    • nominal scale
    • non-normal distribution